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A statistical approach to the zonation of a petroleum reservoir

LeRoy Allan Beghtol

A STATISTICAL APPROACH TO THE ZONATION OF A PETROLEUM RESERVOIR

BY

LeRoy Allan Beghtol

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ABSTRACT

The problem of dividing a reservoir into distinct permeability zones, in order to predict fluid flow and thereby the production history of the field, has long troubled the petroleum engineer. As a solution to this problem a new statistical method for zoning a reservoir was developed, using a modification of the analysis of variance technique. The application of the statistical zonation procedure to permeability data obtained from eight closely spaced wells substantiated the validity and effectiveness of the new method in determining distinct permeability zones.

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INTRODUCTION

One of the major problems in petroleum engineering is that of determining, from existing data, the fluid flow pattern within a reservoir in order to be able to:

1. Predict the production history of the reservoir and the corresponding economics of production.
2. Control, if possible, the efficiency of the production mechanism.

In secondary recovery and pressure maintenance projects the knowledge of the flow pattern is particularly essential. For here, bypassing always occurs and nearly always it is the extent and intensity of bypassing that brings the operation to its economic end.

The data available for interpreting fluid flow comes mainly from coring, where an infinitesimal portion of the reservoir is removed for study from certain wells. Each core sample is analyzed to determine its ability to transmit fluids or its permeability. The resulting permeability values are then commonly plotted against the depth for which they were determined, forming a profile of observations graphically representing the variations found within each cored well.

The problem of predicting fluid flow resolves itself into that of determining the existance, location, and continuity of sedimentarily distinct portions of the reservoir with respect to permeability. The conditions which govern the deposition of sedimentary rocks are such that extreme variations in overall sedimentary conditions are seldom found within a limited area. Distinct zones of permeability are thus found to exhibit some lateral continuity, and it is these zones which must be detected.

From the permeability profiles obtained from each cored well the petroleum engineer postulates the pattern of fluid flow within the reservoir. Presently two methods of forming permeable zones are in common use:

1. Visual inspection of the various profiles and the intuitive construction of continuous permeability zones throughout the reservoir.
2. The construction of capacity distribution curves (plot of permeability versus percent thickness) and the division of the thickness into an arbitrary number of zones.

Both the above methods are based on the assumption that the observed permeability readings extend as continuous bands across the reservoir, and no regard is given as to whether the variations observed represent actual significant differences of simply random variations.

The defects in the existing methods of zonation lead one to search for a more qualitative and quantitative procedure for determining permeable zones. Such a search eventually leads to the field of statistics which deals with the tabulation and presentation of data and the interpretation of data variations. The process of statistical inference which applies to the problem at hand is summarized by Cochran and Cox in their text Experimental Designs: "Variability in results is typical in many branches of experimentation and sampling. Because of this, drawing conclusions from the results is a problem in induction from the sample to the population. The statistical theories of estimation and of testing hypothesis provides solutions to this problem in the form of definite statements that have a known probability of being true."

Statistical methods are herein presented in an effort to formulate a possible means of overcoming the inadequacies of the present methods of zonation. Through the use of a branch of statistics called the analysis of variance, a new zonation procedure will be derived which provides:

1. An efficient and real means of determining distinct permeable zones coupled with a quantitative statement of the probability of such zones being actually distinct.
2. A set of characterizing parameters to serve in correlation.

The main body of this paper is divided into four sections. The first deals with the development and description of the statistical zonation procedure. Section two discusses the assumptions underlying the development of the zonation method. In the third section methods are described by which to test the assumptions of section two. The fourth and final section applies the work of the previous sections to example data.

SECTION A - ZONATION PROCEDURE

A continuous sand body or reservoir can be considered as being composed of k distinct depositional environments, each represented by a mean and a measure of dispersion of some characterizing physical reservoir parameter. Upon considering the complexity of physical reservoir characteristics resulting from deposition during a changing depositional environment, and from subsequent alterations in physical structure due to geologic processes, it would seem logical to assume that within each depositional environment the numerical values of certain reservoir parameters would be more or less randomly distributed. Lateral and vertical gradations within and contamination between these environments are commonly encountered, but will not be considered in this investigation which in itself will be based on a certain amount of isotropy.

Cores obtained from a reservoir sand body may be analyzed to evaluate quantitatively certain physical characteristics of the reservoir at a particular point; vertical variations in these characteristics are due to each of i distinct lithologic environments, where i is equal to or less than k. The problem of detecting, identifying, and correlating existing zones on the basis of some measured physical characteristics, generally permeability, has long been a problem to reservoir engineers.

A statistical method called the analysis of variance has been successfully used to detect significant differences between samples, or sample variations greater than could be expected to occur by chance alone at some preset probability level. The analyses of variance tests of significance are arrived at from the extension to small samples of the theory of least squares as developed by Gauss, Fisher, and others.

It is the object of this paper to apply a modified form of this method to the problem of zonation.

The object in the problem of zonation is to detect the existence of distinct vertical sections or zones within the permeability profile of each well. On the basis that every permeability observation from each cored well belongs to one and only one of i distinct populations with mean u_i and variances σ_i^2 and that Eisenhart's assumptions underlying the analysis of variance (17) are fulfilled, then the standard procedures of the analysis of variance are applicable to the solution of the problem. As the first step, an hypothesis is set up concerning the distribution and uniformity of the data in question; this hypothesis to be tested as to its validity by the analysis of variance method.

Denote a set of measurements by x_{ij}, where x_{ij} is the j^{th} value of x taken from the i^{th} zone. i shall run from 1 to r and j from 1 to n_i. In table form:

Zone Number	Observations
1	x_{11} x_{12} . . . x_{1n_1}
2	x_{21} x_{22} . . . x_{2n_2}
. . .	. . . . . .
r	x_{r1} x_{r2} . . . x_{rn_r}

Let: N = total number of observations

$$x_{\cdot j} = \sum_i x_{ij}/n$$

$$x_{i\cdot} = \sum_j x_{ij}/r$$

$$x_{\cdot\cdot} = \sum_{ij} x_{ij}/N$$

As the hypothesis it will be assumed that the observations x_{ij} from each zone are normally distributed and that the means of these distributions are all equal as are the variances. The total population of the observations is then normal with mean u and variance σ^2 and independent of the zone effect. The method of testing this hypothesis is to determine whether the variability observed in x_{ij} is greater than would be expected under the hypothesis. The design of analysis suited to this task is the single grouping randomized block design, which can be found described in most statistical texts. Table I shows this analysis of variance design as well as the corresponding computational operations for the analysis.

Table I

Analysis of variance table - one variable

Source	Degrees of Freedom	Sum of Squares	Mean Square
Total	rn - 1	$\sum_{ij}(x_{ij} - x_{..})^2$	$\sum_{ij}(x_{ij} - x_{..})^2/(rn-1)$
Between	r - 1	$\sum_{ij}(x_{i.} - x_{..})^2$	$\sum_{ij}(x_{i.} - x_{..})^2/(r-1)$
Within	rn - r	$\sum_{ij}(x_{ij} - x_{i.})^2$	$\sum_{ij}(x_{ij} - x_{..})^2/(rn-r)$

Computational procedure:

Only three values need be calculated in order to determine the required sums of squares.

1. The sum of all observations; $A = \sum_{ij} x_{ij}$
2. The total sum of the squared observations; $S = \sum_{ij} x_{ij}^2$
3. The zone totals; $R_i = \sum_{j} x_{ij}$

The computation of the sum of squares is then as follows:

1. Total sum of squares = $S - A^2/N$
2. Between sum of squares = $\sum R_i^2/n_i - A^2/N$
3. Within sum of squares = $S - \sum R_i^2/n_i$

The ratio of the mean square value for between zones to the mean square value for within zones gives rise to the statistic F which has a known distribution for normal populations and, as such, has been adapted to serve as the test of significance in the analysis of variance procedure. Values of this F-ratio have been tabulated for various probability levels and various degrees of freedom. Two samples are considered to differ significantly if they yield a value of F greater than that given by the table at the chosen level of significance and for the indicated degrees of freedom. The magnitude of the F-ratio bears an inverse functional relationship with both the level of significance and the degrees of freedom. Furthermore, for strict validity in the interpretation of the observed F-value, the samples tested must have equal class numbers or must be weighted accordingly during the analysis of variance. This weighting procedure (10, p. 234) has little effect on the analysis where sample numbers differ by a factor of four or less. As such a factor is seldom exceeded except in exceptionally thick reservoirs, the weighting procedure will not be herein discussed.

Assuming that the group populations are normal or approximately so, nonconfirmation of the original hypothesis by the F-test indicates that either or both the group means and variances are heterogeneous.

If it is assumed that only the zone means are different the analysis of variance model will be $x_{ij} = u + a_i + e_{ij}$; where x_{ij} is the observation on the j^{th} foot within the i^{th} zone whose value depends upon the constant u, or grand well mean, plus a contribution from the zone effect a_i and a local or random error effect e_{ij}. The variates a_i and e_{ij} are assumed to be independent in the probability sense, normally distributed, and with means zero and variance σ^2. Based upon this model it can be shown by the process of expectation that the mean square values from the analysis of variance table are unbiased estimates of the following:

Total mean square = $\sigma^2 + nE\sum_i (u_i - u_.)^2/(rn-1)$

Between mean square = $\sigma^2 + h\sigma_u^2$

Within mean square = σ^2

Where σ^2 is the zone variance

σ_u^2 is the variance between the zone means

$h = (N - \sum_1^r n_i/N)/(r-1)$

E represents the expected value; as $n \rightarrow \infty$

$$E\sum_i (u_i - u_.)^2/(rn-1) \rightarrow \sigma^2 + \sigma_u^2$$

The analysis of variance here used as a test of the homogeneity of means yields, when the hypothesis is rejected, estimates of the components of variance due not only to the variation of zone means but also to variations from all other causes such as random errors, non-normality, unequal zone variances, and the like.

In the preceding pages the basic background for the analysis and testing of well profiles has been discussed. Now it is possible to proceed directly to the development of the procedure for zonation. As a working hypothesis it will be assumed that if a well were divided into r segments, ordered according to depth, such that the variance between segments σ_u^2 would be a maximum while the variance within the segments σ^2 would be a minimum or rather that the ratio of these variances σ_u^2/σ^2 would be a maximum; then these segments will represent the most effective zonation of the well profile.

To measure the effectiveness of the zonation, an index R can be defined to be $\frac{\sigma_u^2}{\sigma^2 + \sigma_u^2}$ such that for a completely uniform reservoir $(\sigma_u^2 = 0)$ R = 0 and that for perfect zonation $(\sigma = 0)$ R = 1. The maximum value of R computed for any profile designates the position of the initial zonal boundary. For computational purposes it is convenient to use the F-ratio or $\frac{\sigma^2 + h\sigma_u^2}{\sigma^2}$ in place of the index R. Furthermore, although the F-ratio is difficultly related to the quantitative effectiveness of the zonation, it has the advantage of providing directly a level of probability for the observed differences between segments. In other words the zonal boundary arrived at by maximizing the F-ratio can be tested to determine for what probability it denotes an actual significant segmentation of the data tested. The number of zones into which a profile must be divided can then be partially determined by the arbitrarily chosen probability level.

The problem of dividing a profile into just two segments so as to obtain the greatest significant difference between the segments is the easiest to solve and as such will be considered first. Let x_1, x_2, . . . x_N be observations which are ordered according with depth from some fixed reference point. For two zones divide the ordered set into two groups for every observation, namely (x_1, x_2, . . . x_n) and (x_{n+1}, x_{n+2}, . . . x_N); and find, as described in the analysis of variance table, the pooled variance within the two groups, the variance within the two groups, and the F-ratio which will determine whether a significant difference exists between the segments. Since the maximum between sum of squares designates the maximum F-ratio, only the former need be calculated in order to determine the position of the zonal boundary. Table 2 illustrates the above procedure by which a profile is broken into two segments for every observation and indicates the additional computational steps needed to determine the between sum of squares (hereafter designated by V.)

Table 2

Division of an ordered set into two groups

n	x_j	$N-n$	x_j
1	x_1	N-1	$x_2 + x_3 + \ldots + x_N$
2	$x_1 + x_2$	N-2	$x_3 + x_4 + \ldots + x_N$
N-1	$x_1 + x_2 + \ldots x_{N-1}$	1	x_N
Segment 1		Segment 2	
$V = (\sum_{1}^{n} x_j)^2/n + (\sum_{n+1}^{N} x_j)^2/(N-n) - T^2/N$			

The maximum V which denotes the zonal boundary is then used in the previous analysis of variance table to determine the F-ratio and the level of significance. The steps in the computation of V as shown in Table 2 are rather complex and can be simplified to those shown in Table 3, which can easily be performed on most hand calculators.

Table 3

Simplified zonation procedure

n	x_j	$n(N-n)/N$	$V = (\sum_{1}^{n} x_j - n\bar{T})^2 \; N/n(N-n)$
1	x_1		
2	$x_1 + x_2$		
N-1	$x_1 + x_2 + \ldots + xN$		

Core recoveries are seldom complete, consequently a complete profile of observations is seldom obtained. In order to assure a complete ordered set of observations, as is necessary in the zonation procedure outlined in Table 2, a method for handling missing data is essential to the analysis; but in the absence of well-to-well correlative information it is impossible to estimate these missing values. The best that can be done is to arrive at a suitable substitute so as to minimize contamination of the profile. Consecutive missing observations should be replaced in such a manner that both the well mean and variance will be unchanged. Fields having large sections of missing data within their well profiles are unsuited for analysis by the statistical zonation procedure as it now stands.

The procedure of zonation herein derived has one serious operational fault which shall hereafter be termed the end effect. If a small rather uniform set of observations, which differ noticeably from the well mean, occurs by chance at either or both ends of the profile, then obviously the variance within said small segment will be low while the variance between will be correspondingly high. Thus a high F-ratio will be found to exist in these regions which would indicate the segregation of a very small number of observations into a distinct zone. This is inconsistent with the intuitive conception of the zonation procedure; moreover, the maximum F-ratio will be unreliable as a test of significance due to the large discrepancy in class numbers between segments.

To conveniently eliminate the more serious of these effects two arbitrary rules should be followed:

1. Only a maximum V-value bounded by significant minimums shall be considered as indicative of a zonal boundary.
2. No zone shall have 'q' or less observations.

Thus any high V-values existing at the ends of a profile due to the end effect will be ignored in the process of zonation with the next highest V designating the position of the zonal boundary. Before continuing, note should be made of the fact that in compensating for the end effect an actual zone of few observations may be ignored. This possibility can be evaluated, however, by noting the position of the occasionally high V-value attributed to the end effect and comparing same with the position of the end effects, if any, in the surrounding wells. The example data used in this paper seems to bear out the acceptability of the above rules. Under Section D this will be more fully discussed and illustrated.

The extension of the zonation procedure from two to three or more zones gives rise to a numerous and complex set of problems. These considerations, however, will be greatly simplified if it is assumed that the first zonal boundary or rather any previously determined zonal division line represents an actual boundary between two zones and will, as such, remain fixed on further subdivision of the profile.

With the first significant zonal boundary assumed to represent an actual division between two zones, continued zonation can then be achieved by applying the two zone method on first one then the other of the previously determined segments. The F-test at the chosen level of significance again serves as indicator for the existence of distinct zonal boundaries. Each newly segregated portion of the profile is further tested by the two zone method until a test of nonsignificance results or until an arbitrary limit is reached with respect to either zone size or number.

It may be argued that the significance of the F-test for more than two zones, as outlined above, cannot be easily related to the efficiency of the zonation; and may in fact lose meaning for only part of the sample is considered when working with more than two zones. Attempts at using the entire sample for testing the significance of the zonation for three or more zones proved so complex and resulted in so little gain in comprehension that they were abandoned.

A brief re-examination of the statistical zonation procedure reveals the following general characteristics:

1. For uniform zones (zero variance within), the zonal boundary will occur where the ratio of the segment means reaches a maximum.

2. For a given variance within, the magnitude of the variance between and the magnitude of the V-value reach a maximum for equal class numbers or zone observations.

SECTION B - UNDERLYING ASSUMPTIONS OF THE ZONATION PROCEDURE

In the first section of this paper numerous assumptions were presented in relation to the development of the zonation procedure. Many of these assumptions require no further explanation, and indeed others lack all but empirical substantiation. However, the underlying assumptions and principles on which the analysis of variance is based are invaluable to an understanding of the process of zonation, with regard to the interpretation of results and require additional enumeration and explanation.

The analysis of variance provides solutions to two different classes of problems:

Class 1: The detection and estimation of fixed relations among means of sub-sets of the universe of objects concerned. Here the parameters involved are means and the issues of interest are the interrelations of the means.

Class 2: The detection and estimation of components of random variation associated with a composite population. Here the parameters are variances and their absolute and relative magnitudes are of primary importance.

The distinction between these two classes must be taken into account in the estimation of the relevant parameters arising from the analysis of variance as well as in the evaluation of the efficiency of the particular experimental design. In particular, the problem of zonation as presented in this paper comes under class 1. Homogeneous variances are assumed, and the problem is to detect whether the observed variations between continuous segments of the profile are significant, coming from different populations, or are such as could occur by chance alone.

The computational steps of the analysis of variance are the same for both classes because the decomposition of the sum of squared deviations of the individual observations from the general mean of the observations into two or more 'sum of squares' is based upon an algebraic identity that is valid whatever the meaning of the numbers involved. When the formulas and procedures are used merely to summarize properties of the data, no assumptions are needed to validate them. However, analysis of variance as a method of statistical inference with respect to the population from which the data is drawn requires that certain assumptions about the population and the sampling procedure must be fulfilled if the inferences are to have meaning. These assumptions, as given by Eisenhart (17), must then first be investigated as to their validity before the preceding algebra of the analysis of variance can be interpreted in the light of existing statistical tests of significance.

Eisenhart's four assumptions underlying the analysis of variance may be briefly stated as follows:

Assumption 1 - Independence of errors: The numbers x_{ij} are random variables that are disributed about true mean values m_{ij}, that are fixed constants. This assumption implies that an unbiased estimator of any linear function of m_{ij} is provided by the same linear function of x_{ij}. Generally, this can be assumed to be the case where proper randomization is assumed as in the sedimentary process where randomization is practically inherent. However, significant correlation of certain physical reservoir parameters with position within the field may be encountered, and to such an extent as to invalidate this assumption. Such cases are generally easily detected and are not applicable to analysis by the present zonation procedure.

Assumption 2 - Additivity of treatment effects: The parameters m_{ij}, are related to the means $m_{i.}$, $m_{.j}$, and $m_{..}$ by $m_{ij} = m_{..} + (m_{i.} - m_{..}) + (m_{.j} - m_{..})$. Thus the differences between any arbitrary pair of column-wise means is a comparative measure of the average difference in effectiveness of the factors identified with these columns. Unless the error variance is small or the treatment effects large, non-additivity will generally be negligible. However if non-additivity is indicated, for instance by the regression of mean on range or mean on variance, it would be well to transform the variables by some suitable method to obtain additivity. Such transforms are discussed in detail by Bartlett (13).

Assumption 3 - Uniformity of variance: The random variables x_{ij} have a common variance and are mutually uncorrelated (zero covariance). The effect of differences between variances will be to reduce the sensitivity of the tests of significance, indicating greater significance than actually exists. The detection of said heterogeneity is difficult, especially so with unknown data.

Assumptions one through three, when valid, allow the use of standard analysis of variance techniques to obtain unbiased estimators of the variances of x_{ij}. This means that an unbiased estimator of the variance of the difference of two observed column means can be evaluated from the residual mean square value. Thus a method for judging whether a real difference exists among the means for the column factors has been developed. The additional assumption of normality gives a quantitative nature to this yardstick of significant differences.

Assumption 4 - Normality: The x_{ij} are jointly distributed in a multivariate normal distribution. When assumptions one through four are satisfied, the analysis of variance procedures for inferring the existence of non-zero differences among population means are valid. As a general rule the effect of non-normality is to make the tests appear more significant than they actually are. Again, it would be better to normalize the data by some suitable transformation; thus providing a more definite solution to the problem.

SECTION C - ANALYSIS OF ZONATION ASSUMPTIONS

The procedure for zonation has significance or meaning only if the preceding assumptions about the data are valid. The validity of assumptions two through four are subject to serious doubt, and of these only Assumption 4 lends itself readily to examination but in so doing casts light upon the validity of the remaining assumptions.

Various graphical and numerical methods exist which can be used to determine or indicate the population distribution of the data thereby examined, and by which the validity of Assumption 4 may be tested. Furthermore, such methods commonly provide additional characterizing parameters which are of potential use in correlation.

A discussion of the more useful of these methods is presented in the following pages; an understanding of which is necessary for their correct use as well as for the correct interpretation of the resulting parameters.

By means of certain characterizing numerical methods, parameters can be determined whose sign and value indicate the underlying geometrical distribution of the population in question. These parameters serve two other purposes; they indicate the amount of the deviation from the normal and the corresponding effect on the statistical tests of significance, and they aid in geologic correlation.

We shall here consider the most common and the most useful of these numerical methods, the method of moments. The r^{th} moment u_r about the mean $\bar{x}$ of the numbers $x_1, x_2, \ldots x_N$ is given by $u_r = (1/N) \sum_{i=1}^{N} (x_i - \bar{x})^r$. Only the first four moments have recognizable significance with respect to direct geometrical interpretation; and they are applicable, with any certainty, only to large samples (≥ 50) or large numbers of samples.

The computational procedures for computing the first four moments are as follows:

$$u_1 = 0$$

$$u_2 = S_2 - S_1^2/N$$

$$u_3 = S_3 - 3S_2S_1/N + 2S_1^3/N$$

$$u_3 = S_4 - 4S_3S_1/N^2 - 3S_1^4/N^3$$

Where N is the number of observations and $S_r = \sum_{i=1}^{N} (x_i)^r$

Using these first four moments as a basis, Fisher (7) developed a set of semi-invariants, termed k-statistics, which serve as unbiased estimators of parameters representing the distribution of the population from which the samples are drawn. These k-statistics are as follows:

$$k_1 = S_1/N$$

$$k_2 = (NS_2 - S_1^2)/N(N-1)(N-2)$$

$$k_3 = (N^2S_3 - 3NS_2S_1 + 2S_1^3)/N(N-1)(N-2)$$

$$k_4 = \left[N(N+1)S_4 - 3(N-1)S_2^2\right]/(N-1)(N-2)(N-3)$$

The first two k-statistics are respectively the sample mean and the unbiased estimator of the population variance.

k_3, termed the third cumulant, serves as a measure of skewness or assymetry; its magnitude indicating the amount by which the more strongly developed observations deviate from the position of the mean, and its sign indicating the direction of this deviation. Maximum development to the left of the mean is taken as positive, while development to the right is negative. For practical purposes a new statistic g_1 is determined so that it is independent of the original units of the data and therefore applicable as a comparison to measurements of different types and magnitudes, $g_1 = k_3/K_2^{3/2}$.

The fourth cumulant K_4 forms the basis for the measure of kurtosis, or the contamination of an otherwise normal distribution. It reflects by its sign a greater (positive) or fewer (negative) number of large deviations than would be expected were the distribution normal. Again a dimensionless statistic g_2 is used as the direct measure of kurtosis, where $g_2 = k_4/k_2^2$. The extreme sensitivity of g_2, due to the inclusion of the fourth power of the variate, seriously affects its applicability in the analysis of widely variable and often highly contaminated permeability data. Only with the largest of samples (>100) should the algebraic complexity of the computation of g_2 be undertaken.

For comparisons with normal distributions, tables have been constructed, showing the expected values of g_1 and g_2 for various probabilities and samples sizes. Such a table has been computed by Bennet and Franklin (1, p. 95). The most commonly encountered numbers of observations were used and the 5% and 1% levels of significance were chosen. Normally distributed data should have values of g_1 and g_2 which exceed those in the table only 5% and 1% of the time. If these values for 5% are exceeded, the normality of the underlying distribution would be questionable.

Numerous graphical procedures are available for the purpose of depicting the underlying distribution of a set of observations. They, however, are by no means equally efficient or applicable as will be seen from the following discussion of the more pertinent of the methods.

Tabulation into classes and the graphical presentation of this data as histograms is commonly used for rough preliminary visual appraisal of the distribution. However with small numbers of widely variable data, the descriptive ability of histograms is a sensitive function of the arbitrarily chosen class interval; and, as such, may not only fail to disclose pertinent information but may even suggest erronous conclusions. Furthermore, raw histogram data is not always reliable or indeed applicable for direct statistical interpretation. Only by successive trial and error can a class interval be found such that the underlying geometry is clearly pictured. Although arbitrary rules cannot be set down for choosing appropriate class intervals, the maximum range is of prime importance in affecting this choice. Generally division of the range into ten to twenty-five intervals will be sufficient for most problems.

Histograms are of use only for tentative initial insights into the geometrical shape of the data distribution. More exact work demands a more quantitative graphical method such as is found in the cumulative percent curve.

The cumulative percent curve is formed by:

1. Ranking the observed data in the order of their increasing magnitude.
2. Obtaining the cumulative sum of this ranked data.
3. Determining the cumulative percent corresponding to each ranked observation.

The resultant plot of cumulative percent versus the magnitude of the corresponding observation establishes the desired curve. The ordinate (P) for any abscissa (x) gives the percent of the distribution having values less than or equal to that particular abscissa. Now only a finite number of observations are available for plotting this curve, but for practical purposes these points outline the continuous curve that would result from infinite sampling. From this intuitive assumption of continuity one is free to determine values other than those actually recorded.

In the cumulative percent curve is found a detailed quantitative means of portraying the sample distribution which is independent of the units, as long as they are linearly related, or the numerical magnitude of the data involved. The median of the distribution is determined by the abscissa corresponding to the 50th percentile, and the dispersion is indicated by the inter-quartile range ($x_{75\%} - x_{25\%}$). For a normal curve σ = 3/4 of the inter-quartile range.

Transferring the cumulative percent curve to probability paper offers a convenient means of determining the departure of the distribution from the normal, and provides a means for rapidly checking any postulated distribution by applying the corresponding normalizing transform and visually observing the resultant plot. Probability paper is so designed that any normal probability distribution yields a straight line when plotted on this paper. From this straight line two parameters, the mean and the standard deviation, can be found which completely characterize the distribution. The mean is the value on the abscissa corresponding to the 50% fractile ($x_{50\%}$). The standard deviation must be obtained from two points along the curve. For the normal population it can be shown mathematically that approximately 68.27% of the distribution is included within the range $\bar{x} \pm \sigma$. Therefore a graphical estimate of the standard deviation can be obtained from the difference of the abscissa values corresponding to the 50% and either the 15.9% or 84.1% fractiles.

SECTION D - ANALYSIS OF EXAMPLE DATA

The data used to test and exemplify the procedures previously outlined was obtained from eight wells cored into a fairly continuous horizontal fine grained micaceous sandstone of Pennsylvanian age, averaging 55′ in thickness, at a depth of approximately 1650′, and occurring within the Olympic Pool, Hughes County, Oklahoma. These eight wells cover approximately a quarter of a section in area and roughly follow the northeast-southwest trend of the sand section; their planner relationship being shown in Figure 1.

The reservoir parameter analyzed was permeability; and the data consists of such permeability observations, one for every cored foot of the sand section. Inch plugs were removed from the well cores at foot intervals, and the absolute permeabilities of these plugs were then measured and recorded, with an accompanying graphical representation, as shown in Appendix A. Observations from the eight wells range from 0 to 337 millidarcys; the mean of the observations is 30 millidarcys; and the majority of the high values appear to occur in samples from the top of the profiles.

To facilitate the presentation of this section, the operational steps employed will be discussed with respect to only one example well (Well K-17) for which the indicated calculations have been performed and recorded in Appendix B.

Initially an attempt was made to characterize the distribution of the permeability observations as found within the given section. Percent histograms were prepared from each well profile. The observations forming the well profiles were arranged in order of increasing magnitude. A convenient class interval was then chosen by inspection and the percent of the observations found in each value range was recorded and plotted against the corresponding value for each range interval. Section 1-B and Figure 1-B of Appendix B illustrate the ordered data involved and the resulting histogram obtained for Profile K-17. The geometrical shape represented by this histogram is very similar to those obtained from every other well. The marked skewness to the left and the long drawn-out tail in the region of high values is typical of a logarithmic distribution, or one in which the logarithm of the data is normally distributed. As this indicated distribution might be the result of two or more superimposed distributions, each of uncertain nature, it cannot be immediately interpreted as characterizing the distribution within each depositional environment.

To obtain a further insight into the distribution of the data, each profile was divided into two segments by the statistical zonation procedure; and each segment was then examined by both numerical and graphical methods. Table 4 illustrates the permeability means and standard deviations corresponding to the eight wells and their respective two subdivisions. Percent histograms prepared for each segment were similar to each other as well as to those previously prepared for the wells as a whole (See Appendix B; Section 1, Figure 2-B). The method of moments indicates positive skewness and kurtosis; both of which vary significantly from the normal (See Appendix B; Section 2-B). From the similarity of histogram geometry and parameter values produced from each well and each well segment, it is assumed that the permeability observations under consideration have a common type distribution.

To test the apparent similarity of this type distribution to the lognormal, a cumulative percent curve for each well and each well segment was prepared, as shown in Section 1-B of Appendix B, and plotted on logarithmic probability paper (probability paper with the abscissa in logarithmic spacing). The resulting approximate straight lines would seem to confirm the hypothesis that the data studied is distributes as the lognormal.

Figure 3-B of Appendix B shows the curves obtained from the plot of the cumulative percent data for well profile K-17 on probability and log probability paper. Both curves differ from straight lines, but the one on log probability paper less so. Confirmation of the hypothesis of a lognormal distribution is not evident until the cumulative percent curves for the two determined segments of profile K-17 are plotted on log probability paper (Appendix B; Figure 4-B). Segment 2 gives a straight line. Segment 1, however, appears to be composed of two dis-tinch straight line portions. Later in this section it will be shown that the observations forming each straight line portion of Segment 1 actually come from two distinct zones. Similar relations observed when working with the other profiles indicate that a rough visual method for determining the number of zones might be to count the number of major in-flection points of the cumulative percent curve on log probability paper.

From the straight lines observed when the cumulative percent curve is plotted on log probability paper, both the mean and standard deviation of the transformed and the original data may be directly approximated. The mean of the transformed data equals the logarithm of the abscissa corresponding to the 50% fractile - $M\{\log x\} = \log x_{50\%}$. The standard deviation of this distribution is given by:

$$\sigma\{\log x\} = (\log x_{50\%} - \log x_{15.94}) = (\log x_{84.1\%} - \log x_{50\%}) = \sigma_L$$

Mathematically the relation of the transformed data to the original is given by the following equations:

$$\log \bar{x} = \log x_{50\%} + 1.1513\, V_L^2$$

$$V_L = 0.4343 \frac{\sigma\{x\}}{\bar{x}} \quad ; \text{(only when } V_L \text{ is small)}$$

The constants here indicated are for the logarithm to the base ten. For practical purposes it has been demonstrated that both log x and x can be regarded as normally distributed as long as the ratio of the standard deviation to the mean for x is less than 1/3 or as long as the standard deviation of log x is less than 0.14.

It may now be reasonably assumed that the permeability observations under consideration are distributed as the lognormal having marked positive skewness, with the mode to the left of the mean and a long drawn-out tail towards the right high-valued side. The distribution is furthermore more sharply peaked than the normal, being leptokurtic (positive kurtosis).

Referring to table 4 it is apparent that there is a definite correspondence between the means and standard deviations for the wells and well segments. Mean and standard deviation seem to bear almost a one to one relation to each other which invalidates Assumption 2. Transforming the data to their respective logarithms is the commonly applied technique used to stabilize these two parameters in such cases. Thus the logarithmic transform of the permeability observations would seemingly result in a set of data tentatively satisfying both Assumptions 2 and 4 and suited to zonation by analysis of variance techniques.

The proceeding analysis indicates that raw permeability data is unsuited to zonation by the method herein derived; therefore, the data in the well profiles was transformed to their conresponding logarithms to the base ten, zero values being recorded for the logarithm of permeability observations less than or equal to one. The transformed data is shown in Appendix A with a corresponding graphical representation. The zonation procedure was then applied to each well profile of transformed data until no significant division line between segments could be detected. Section 3-B of Appendix B illustrates this procedure.

Here division of the 37 observation profile is at the 11th foot, as indicated by the maximum V-value bounded by distinct minimums occurring there. The high V-value occurring at the 36th foot illustrates the end effect. The zero value of the 37th observation, being greatly different from the mean of 1.050, results in a high variance between segments and a correspondingly high V-value as indicated. The analysis of variance when applied to the two indicated segments results in a F-ratio of 9.067 for 1 and 36 degrees of freedom. The table of F-values for these degrees of freedom gives a value of 7.39 at the 1% level. As this value is less than that calculated, a significance of greater than 99% is indicated.

Continued zonation of the two determined segments produced the following results:

For segment one, the maximum V-value indicated division at the 6th observation. Inclusion of this value in the analysis of variance table produced an F-ratio of 3.48 for 1 and 11 degrees of freedom. This value is less than that given by the F-table for the 5% level, 4.48, and as such was taken to indicate a nonsignificant difference between segments.

Segment two was found to give a maximum V-value at the 25th observation due to the previously encountered zero end value. Ignoring this end effect, the next highest V indicated division of the segment at the 11th observation. The corresponding F-ratio of 6.73 fell between the values in the F-table at the 5% and 1% levels, 4.24 and 7.77 respectively, for the indicated degrees of freedom. Further examination placed the level of significance around 2%, or the chances are 2 out of a hundred that the observed difference between segments could have occurred by chance alone.

Continued zonation of the newly determined segments resulted in tests of nonsignificance. Well K-17 then has three significantly different segments, which shall be considered as representing three distinct depositional environments or permeability zones.

The zonal boundaries determined for all eight wells are shown by horizontal red lines on each well profile of Appendix A. High values of V resulting from the end effect are also indicated in red, being labeled EE. On comparing the profiles of both the original and the transformed data as well as the positions of the zonal boundaries, it would seem that no inconsistancies are present and that the zonation of the transformed data corresponds to an intuitive sense of rightness. Furthermore, the indication is that a zonal boundary detected using the original data will also be detected some time during the zoning of the transformed data, though not necessarily equally significant or detected in any particular order.

Table 5 represents a tabulation of the means and standard deviations of the observations corresponding to each well and each distinct well segment of transformed data. It is apparent from this table that the means and standard deviations are independent. Moreover, as the standard deviations are all approximately of the same order of magnitude, Assumption 3 would seem to be sufficiently satisfied for this analysis. Note should be made of the fact, however, that as differences in variances do exist between some segments of some profiles they no doubt contribute to the significant segregation of those profiles.

After having zoned each well profile, it becomes necessary in the study of the fluid flow pattern to extend the segments of each profile across the reservoir to form continuous zones, that is to correlate zones of comparable permeability well-to-well across the field. If a segment of some profile was determined to be not significantly different from the remainder of the profile; and yet a zone of mean and standard deviation corresponding to this segment was significantly detected in the surrounding wells, then it might be well to call this nonsignificant segment (based on one well) part of the surrounding zone. However, a simplified procedure for performing such an operation is lacking; and at best all that can be done is to record the nonsignificant zonal boundaries determined in an effort to intuitively carry out said operation.

With the example problem at hand only the significantly segmented portions of each profile were used to form zones, thus avoiding complexity and ambiguity in the following discussion. As the means of the segments from the eight wells of the field are noticeably different, they were used as the parameters for correlation across the field. Segments of similar means were sequentially connected to form the main zones of the sand section as shown in the geologic cross section pictured in Figure 2. Only three continuous zones are apparent, and these were labeled zones 1, 2, and 3. For these, the block type representative of Figure 3 conveniently presents the characteristics of the samples composing each major zone and immediately lends itself for analysis as to the existence of trends and extreme or nonsignificant values.

A randomized block design of the type shown in Appendix C can be used to examine each constructed major zone for trends or significant variations within the zones, as represented by significant row and column variations or interactions. The results of such an analysis can be found in Appendix D and are here summarized:

Zone 1: This upper zone is found in only four of the eight wells, being chiefly represented in the northwest row. It is of rather uniform thickness and bears only an indefinite relation to the lower zones. Statistically the contribution of well 1-Q-11 is significantly different from those of the remaining wells, as determined by the t-test. The calculation of the missing values necessary to complete the block design was not performed because of insufficient data upon which to base such calculations.

If as indicated observation 1-Q-11 is ignored as not belonging to Zone 1, then the remaining three observations form a rather uniform zone as indicated by the small standard deviation of 0.0388. Otherwise both significant row and column differences are indicated.

Zone 2: This middle zone of the reservoir is found in all but well M-11 and represents the most continuous as well as one of the most uniform zones detected. Zone 2 rests directly on Zone 3 and thickens noticeably in the south. With the calculated missing value, the analysis of variance was unable to detect any significant variations within the zone. The low standard deviation of 0.0825 further attests to the uniformity of this zone.

Zone 3: The northeastern portion of the reservoir contains this lower zone which is represented in five of the eight wells. Thickening occurs in the southwest, and the analysis of variance shows no significant trends. The high standard deviation of 0.4583, however, indicated substantial heterogeneity within the zone.

Assuming that the average permeability within the three major zones detected will substantially control fluid flow, what value should be used to represent the average overall zone permeability? Commonly the arithmetic average of the raw data is used in this respect, but such an average can only have meaning if the permeability readings obtained through coring extend as continuous bands throughout each zone. Comparisons of these mean values with those obtained from actual flow tests at the well, properly modified for extraneous influences, have often proved how unsatisfactorily the arithmetic mean represents overall zone permeability. Indeed, one of the initial assumptions in developing the zonation procedure was that of random distribution of permeability values throughout each zone.

If we consider only a linear horizontal path through any zone, fluid will flow along this path from one permeability increment to another; the value of such increments being indicated by the magnitude of the observations within each zone profile and the variability in magnitude of such increments being indicated by the zone standard deviation. In such a case the path permeability would be characterized by the harmonic mean of the data in the zone profile. However, flow of fluids through a sand reservoir is not necessarily linear, the fluid seeking the path of least resistance. The harmonic mean is then too small, for it greatly diminishes the effects of large values which definitely play an important role in controlling fluid flow. The geometric mean gives more weight to these large values and yet not full weight as does the arithmetic mean, and as such, is, in the author's opinion, a more suitable mean by which to represent the average zone permeability.

The antilogarithm of the sum of the logarithms of the permeability observations divided by the total number of observations represents the geometric mean of the data and is readily available from previous calculations. For the three major zones these values are as follows:

Zone 1 - 59.49 md

Zone 2 - 23.29 md

Zone 3 - 11.65 md

Obviously the lower the standard deviation of the data within each zone the more uniform will be the flow and the more closely will the geometric mean represent the zone permeability. Further work to determine a parameter capable of accurately representing the overall zone permeability is vitally needed for the more effective evaluation of fluid flow within the reservoir.

CONCLUSIONS

1. The zonation procedure as developed in the body of this paper appears to represent a very useful tool for the detection of distinct segments within any ordered set of data, as demonstrated by the results obtained with the example data. From the initial assumptions underlying the construction of said zonation procedure, the efficiency or effectiveness of the method is high and as such represents an improvement over existing zonation techniques. Furthermore, the procedure enables independent analysts to arrive at the same results, provides additional means or parameters for correlation, and is easily programmed for use on the standard digital computers to simplify the work of zonation.
2. The permeability observations for the field in question have an approximate lognormal distribution, or in other words the logarithms of the observations are themselves normally distributed. Random high values are common within the data and mask the existence of distinct depositional environments. Indications are that most permeability observations are distributed as the lognormal, confirming previous work of other analysts, and as such should be transformed to logarithms prior to analysis.
3. A single parameter representing the average overall permeability of any zone would seem to be best found in the geometric mean, which is directly obtainable from the zonal computations performed upon the logarithmic transformed data.

RECOMMENDATIONS

1. The quantitative effects of nonvalidity of the assumptions underlying the fabrication of the zonation procedure should be investigated. At present only empirical methods seem to be available for such an investigation, and at best only broad generalized statements concerning these effects could be hoped for.
2. The effects of unequal segment numbers upon the significance of the F-test should be further determined, and general rules should be established for treating such unequal class numbers.
3. Further work should be performed to determine the most representative parameter for identifying the overall zone permeable capacity. Perhaps the application of multiple correlation analysis to core data and field tests of well productivity would provide an answer to the question.

APPENDIX A

Tabulation and Graphical Representation of Absolute Permeability Data, Both Regular and Logarithmic, For Eight Sample Wells

Notation:

Perm. - Permeability, here measured in millidarcys (md.).

Log - Logarithm to the base ten.

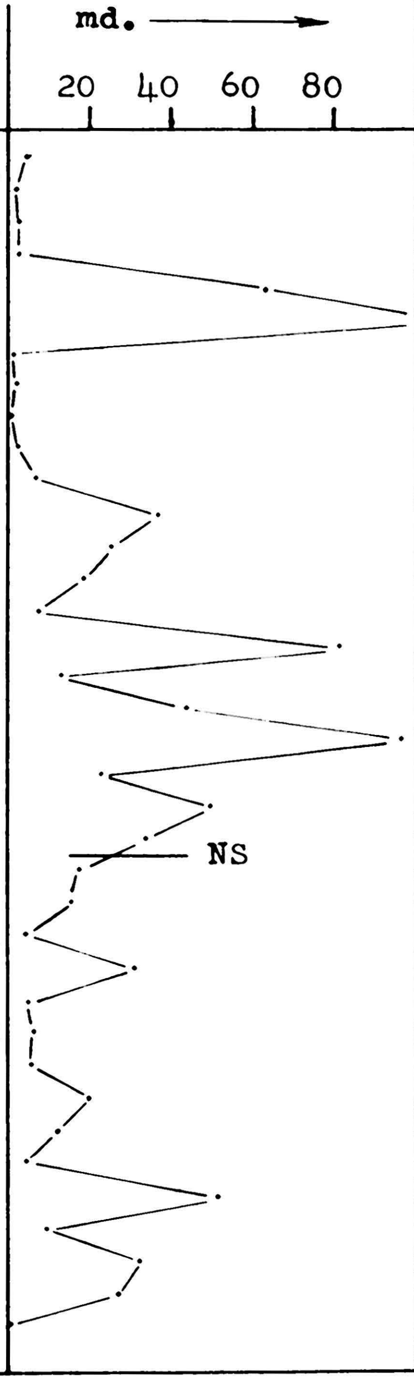
Horizontal red line - Position of determined zonal boundary; labeled alphabetically in order of determination with level of significance indicated.

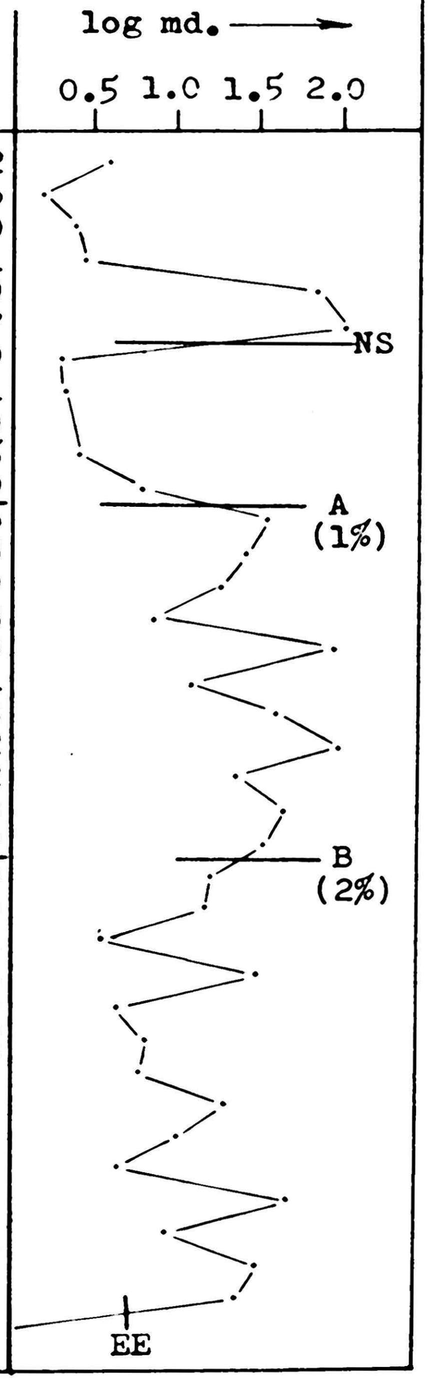
Horizontal black line (NS) - Position of nonsignificant determined zonal boundary.

Vertical red line (EE) - Position of high V-value attributed to the end effect.

Well K-17

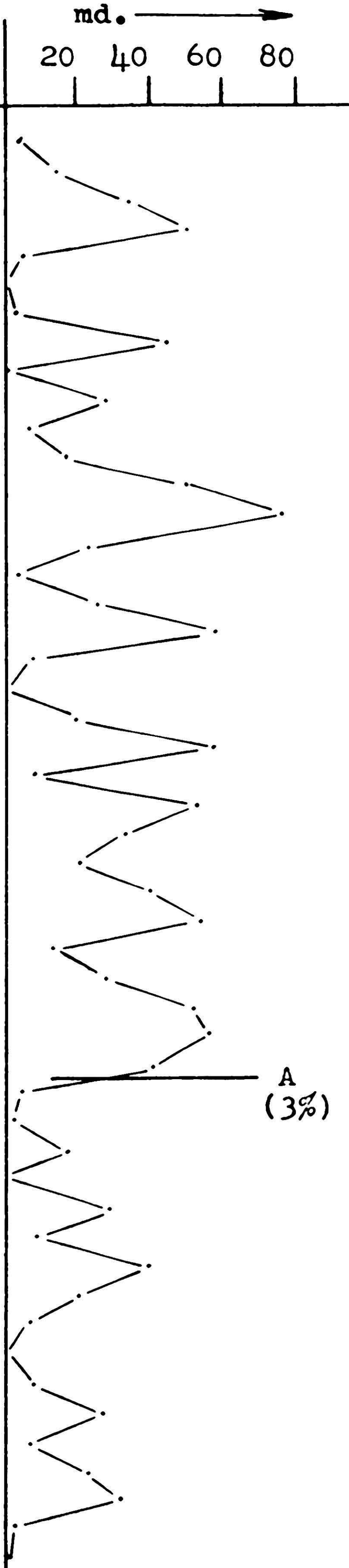
Depth (ft)	Perm.	Log Perm.
1658	4	0.602
1659	1.5	0.176
1660	2.4	0.380
1661	2.7	0.431
1662	63	1.799
1663	109	2.037
1664	1.9	0.279
1665	2.0	0.301
1666	0.9	0.000
1667	2.6	0.415
1668	6.3	0.799
1669	36	1.556
1670	25	1.398
1671	19	1.279
1672	6.8	0.833
1673	81	1.908
1674	13	1.114
1675	43	1.633
1676	96	1.982
1677	23	1.362
1678	49	1.690
1679	33	1.519
1680	17	1.230
1681	15	1.176
1682	3.4	0.531
1683	31	1.491
1684	4.9	0.690
1685	6.2	0.792
1686	6.1	0.785
1687	20	1.301
1688	12	1.079
1689	4.8	0.681
1690	51	1.708
1691	9.3	0.968
1692	32	1.505
1693	27	1.431
1694	0.8	0.000

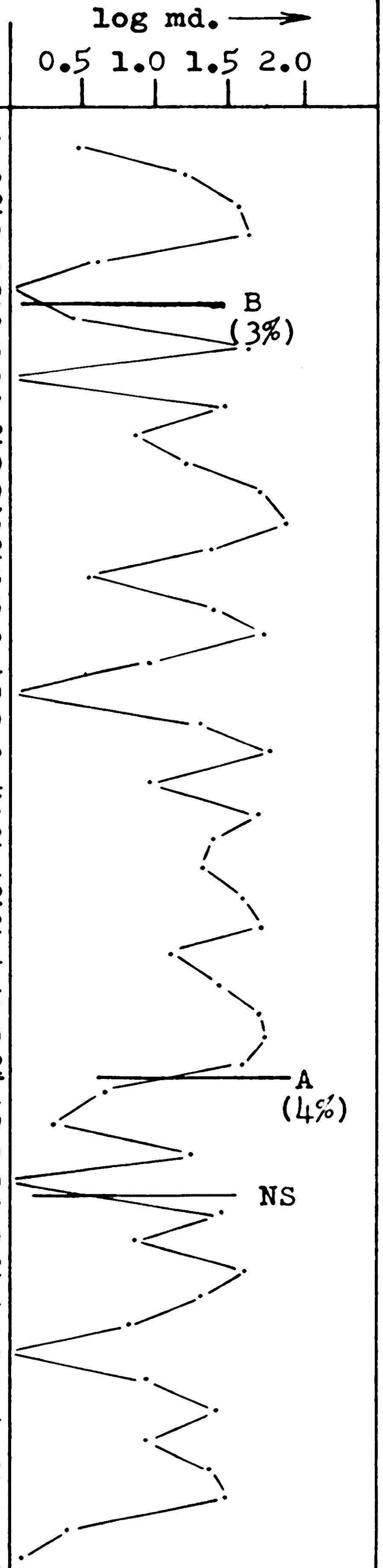




Well K-15

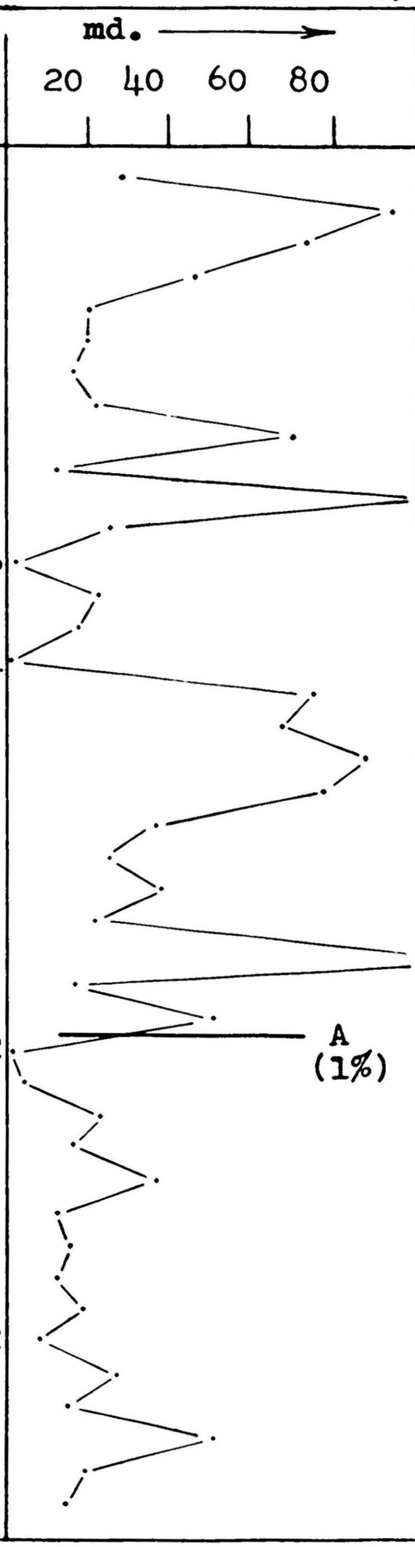
Depth (ft)	Perm.	Log Perm.
1648	2.8	0.447
1649	14	1.146
1650	34	1.532
1651	50	1.699
1652	4.2	0.623
1653	0.0	0.000
1654	3.0	0.466
1655	45	1.653
1656	0.6	0.000
1657	28	1.447
1658	6.5	0.812
1659	17	1.230
1660	50	1.699
1661	75	1.875
1662	23	1.362
1663	3.7	0.568
1664	25	1.398
1665	57	1.756
1666	7.8	0.944
1667	0.3	0.000
1668	19	1.279
1669	57	1.756
1670	8.8	0.944
1671	52	1.715
1672	23	1.362
1673	20	1.301
1674	40	1.602
1675	54	1.732
1676	13	1.114
1677	27	1.431
1678	51	1.707
1679	55	1.740
1680	40	1.602
1681	4.8	0.681
1682	1.9	0.279
1683	17	1.230
1684	0.1	0.000
1685	28	1.447
1686	7.4	0.869
1687	40	1.602
1688	20	1.301
1689	6.5	0.813
1690	0.5	0.000
1691	8.7	0.940
1692	26	1.414
1693	8.1	0.908
1694	21	1.322
1695	31	1.491
1696	2.5	0.398
1697	1.3	0.114

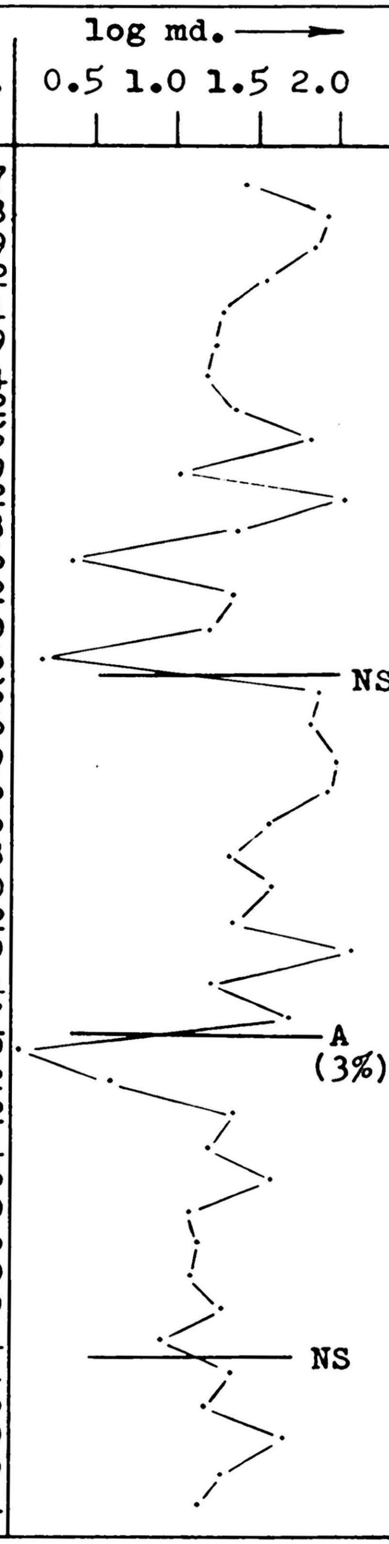




Well M-15

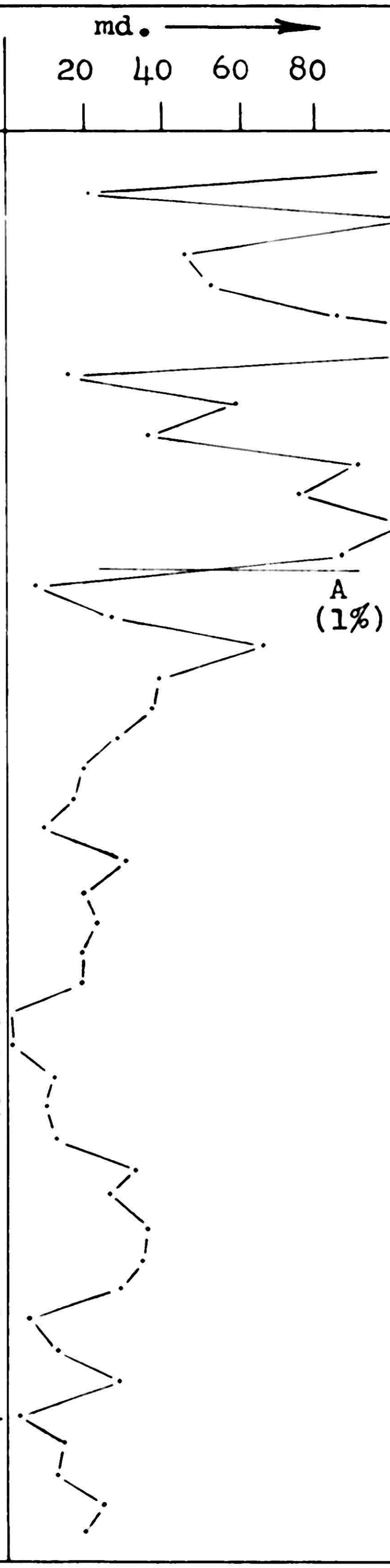
Depth (ft)	Perm.	Log Perm.
1653	28	1.447
1654	94	1.973
1655	74	1.869
1656	46	1.662
1657	20	1.301
1658	19	1.279
1659	16	1.204
1660	21	1.322
1661	70	1.845
1662	12	1.079
1663	103	2.012
1664	25	1.398
1665	3.6	0.356
1666	22	1.342
1667	17	1.230
1668	1.4	0.146
1669	75	1.875
1670	67	1.826
1671	87	1.939
1672	77	1.886
1673	36	1.556
1674	25	1.398
1675	38	1.580
1676	21	1.322
1677	120	2.079
1678	16	1.204
1679	51	1.707
1680	1.2	0.079
1681	4.0	0.602
1682	23	1.362
1683	16	1.204
1684	36	1.556
1685	12	1.079
1686	15	1.176
1687	12	1.079
1688	19	1.279
1689	8.2	0.914
1690	27	1.431
1691	15	1.176
1692	50	1.699
1693	19	1.269
1694	13	1.114

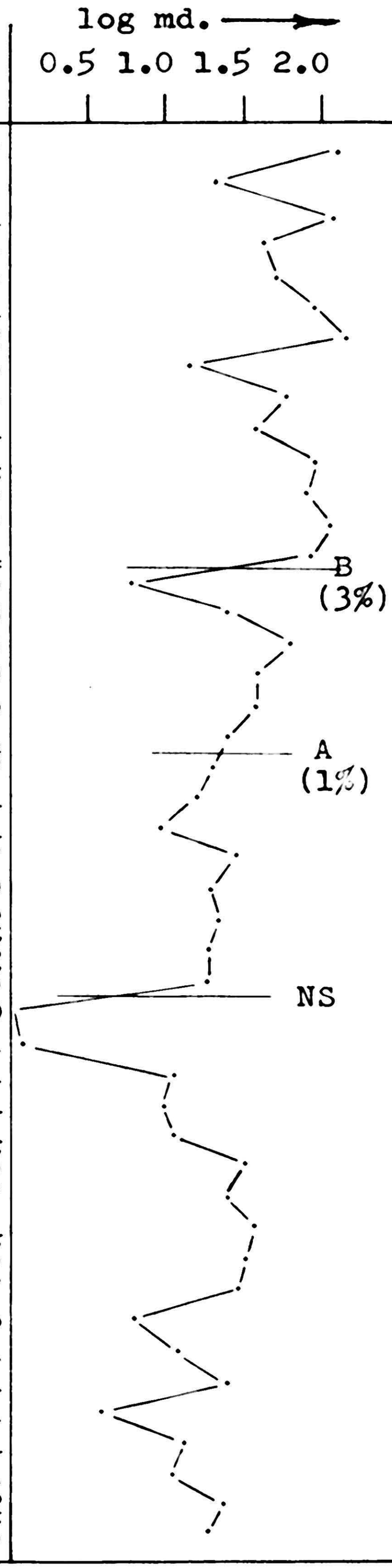




Well K-13

Depth (ft)	Perm.	Log Perm.
1652	131	2.117
1653	20	1.301
1654	127	2.104
1655	46	1.663
1656	52	1.716
1657	86	1.934
1658	142	2.152
1659	15	1.176
1660	58	1.763
1661	35	1.544
1662	90	1.954
1663	75	1.875
1664	112	2.049
1665	86	1.934
1666	5.7	0.756
1667	25	1.398
1668	65	1.813
1669	38	1.580
1670	36	1.556
1671	27	1.431
1672	19	1.279
1673	16	1.204
1674	8.8	0.944
1675	30	1.477
1676	19	1.279
1677	22	1.342
1678	18	1.255
1679	18	1.255
1680	0.1	0.000
1681	1.1	0.041
1682	11	1.041
1683	8.2	0.914
1684	11	1.014
1685	32	1.505
1686	25	1.447
1687	35	1.544
1689	32	1.505
1690	28	1.447
1691	5.7	0.756
1692	11	1.041
1693	27	1.431
1694	3.4	0.531
1695	13	1.114
1696	11	1.041
1697	23	1.362
1698	18	1.255





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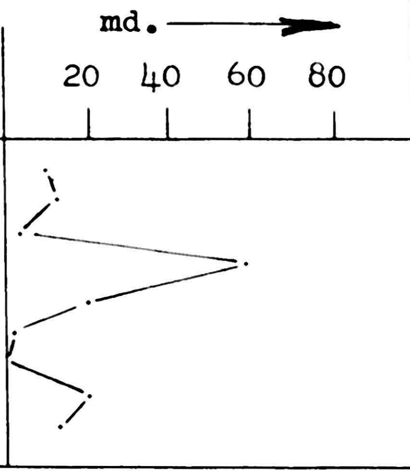
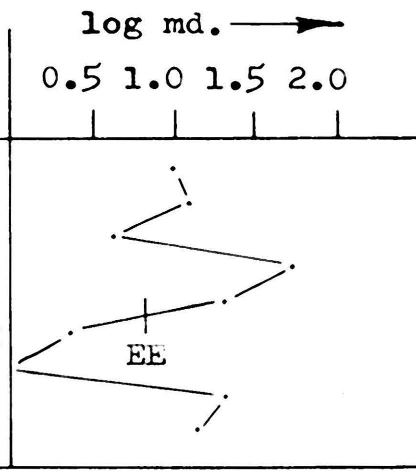
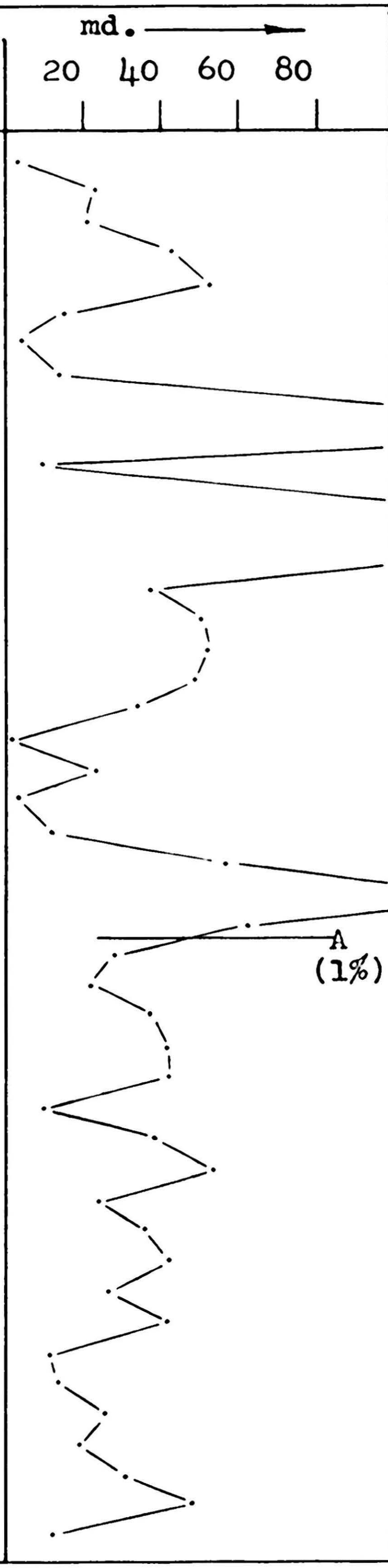
Depth (ft)	Perm.	Log Perm.
1699	9.7	0.987
1700	12	1.079
1701	3.8	0.580
1702	58	1.763
1703	19	1.279
1704	2.3	0.362
1705	0.1	0.000
1706	19	1.279
1707	12	1.079

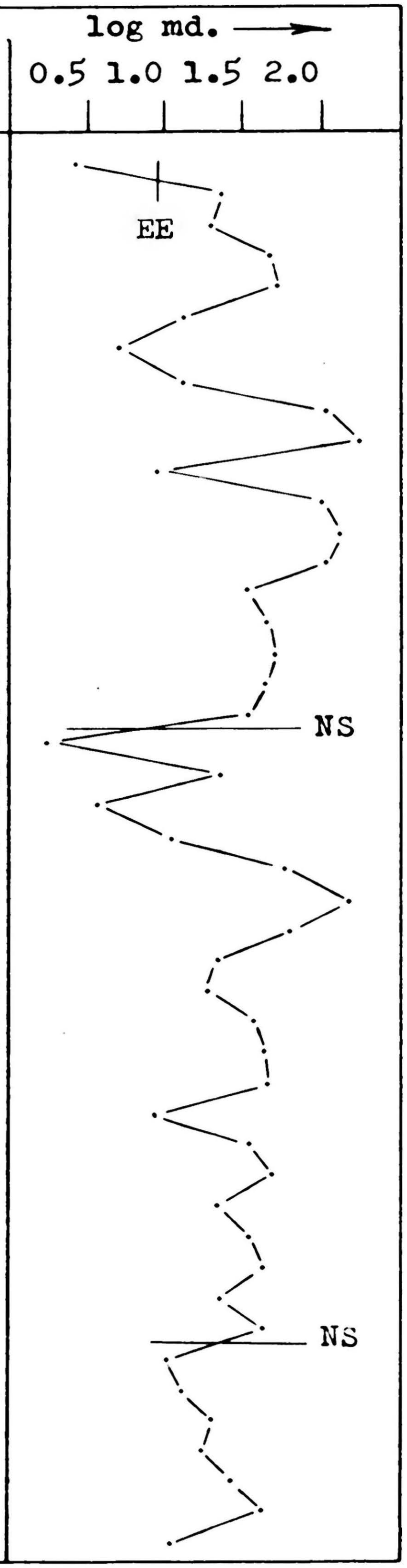




Well 1-0-13

Depth (ft)	Perm.	Log Perm.
1669	2.6	0.414
1670	23	1.380
1671	20	1.301
1672	43	1.633
1673	53	1.724
1674	14	1.146
1675	5.5	0.740
1676	13	1.113
1677	120	2.079
1678	173	2.238
1679	9.0	0.954
1680	102	2.008
1681	138	2.140
1682	107	2.029
1683	37	1.568
1684	50	1.699
1685	52	1.716
1686	49	1.690
1687	34	1.532
1688	1.7	0.230
1689	23	1.362
1690	3.6	0.556
1691	11	1.041
1692	57	1.756
1693	175	2.243
1694	63	1.799
1695	28	1.447
1696	22	1.342
1697	37	1.568
1698	41	1.612
1699	42	1.623
1700	9.5	0.978
1701	38	1.580
1702	54	1.732
1703	24	1.390
1704	36	1.556
1705	42	1.623
1706	26	1.432
1707	42	1.623
1708	11	1.041
1709	13	1.114
1710	25	1.398
1711	19	1.278
1712	30	1.477
1713	48	1.681
1714	12	1.079





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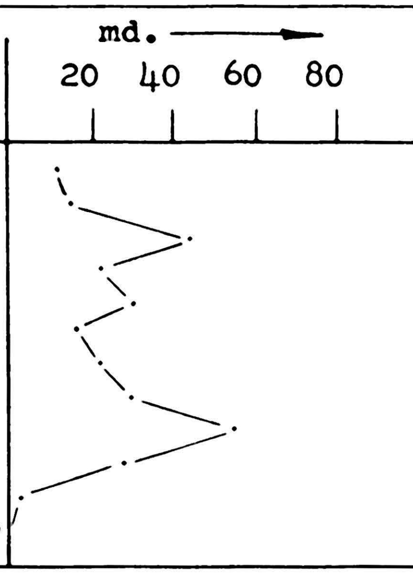
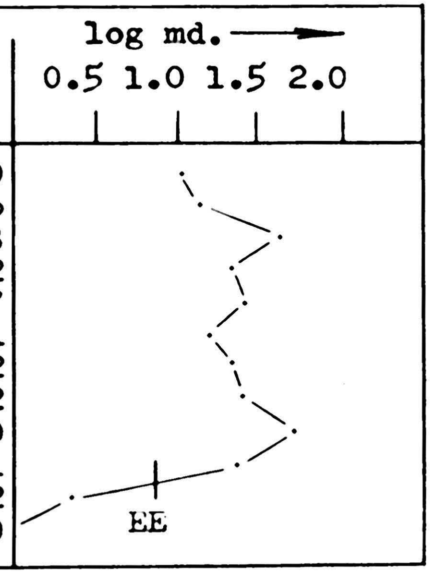
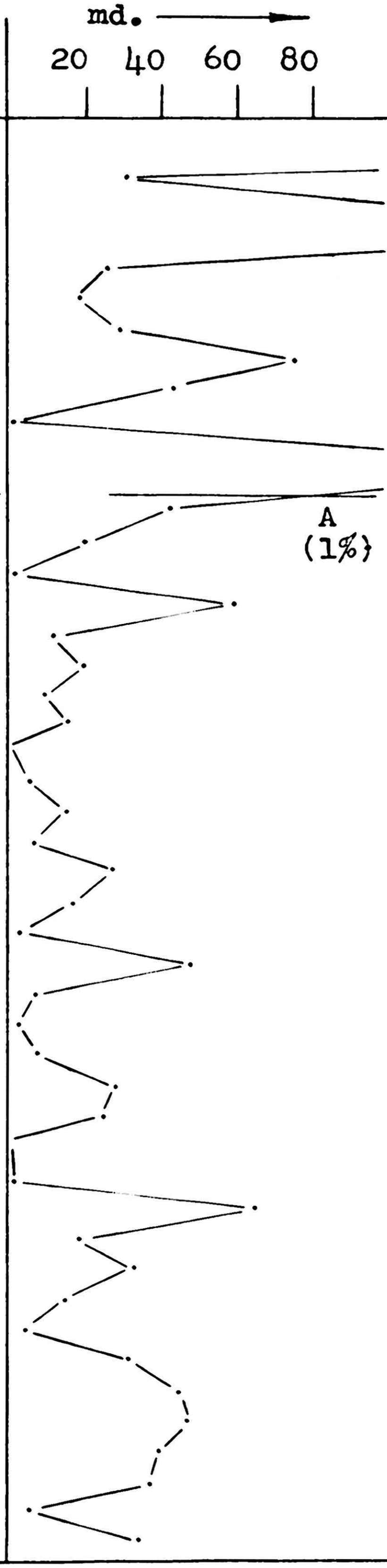
Depth (ft)	Perm.	Log Perm.
1715	12	1.079
1716	15	1.176
1717	44	1.643
1718	22	1.342
1719	30	1.477
1720	16	1.204
1721	22	1.342
1722	29	1.462
1723	55	1.740
1724	27	1.431
1725	2.3	0.362
1726	0.6	0.000

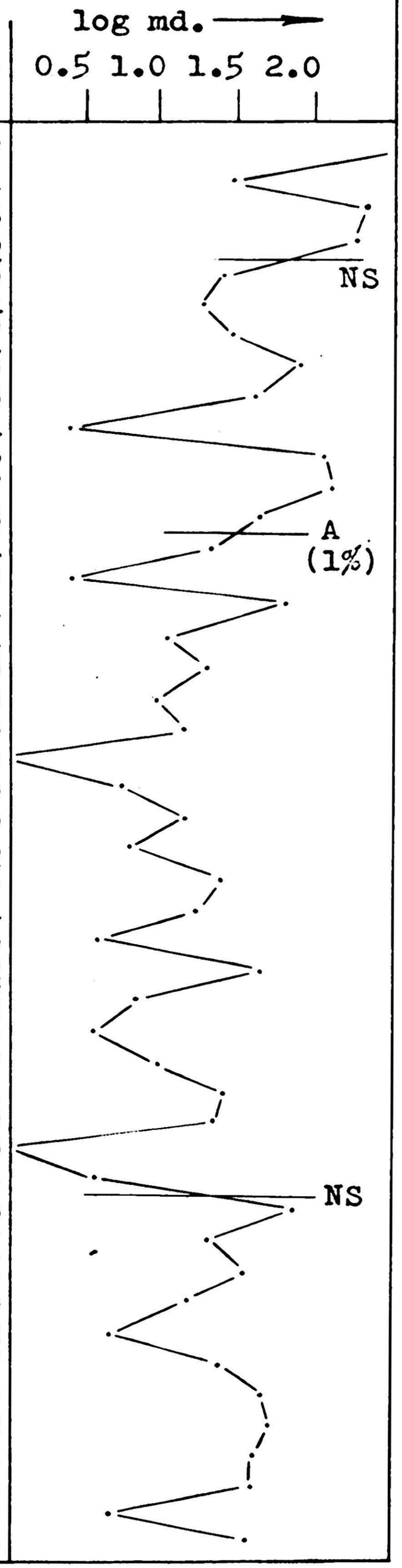




Well M-11

Depth (ft)	Perm.	Log Perm.
1706	337	2.527
1707	31	1.491
1708	247	2.393
1709	216	2.336
1710	26	1.415
1711	18	1.254
1712	28	1.447
1713	75	1.875
1714	43	1.633
1715	2.0	0.301
1716	104	2.016
1717	128	2.107
1718	42	1.623
1719	20	1.301
1720	2.4	0.380
1721	59	1.771
1722	11	1.041
1723	19	1.278
1724	9.5	0.978
1725	15	1.176
1726	0.2	0.000
1727	5.2	0.716
1728	14	1.146
1729	5.7	0.756
1730	26	1.415
1731	16	1.204
1732	3.6	0.556
1733	47	1.672
1734	6.1	0.785
1735	3.2	0.505
1736	8.5	0.929
1737	27	1.431
1738	24	1.380
1739	0.9	0.000
1740	3.3	0.518
1741	64	1.806
1742	19	1.297
1743	32	1.505
1744	15	1.176
1745	4.7	0.672
1746	30	1.477
1747	44	1.643
1748	46	1.662
1749	39	1.591
1750	37	1.568
1751	5.0	0.699
1752	33	1.518





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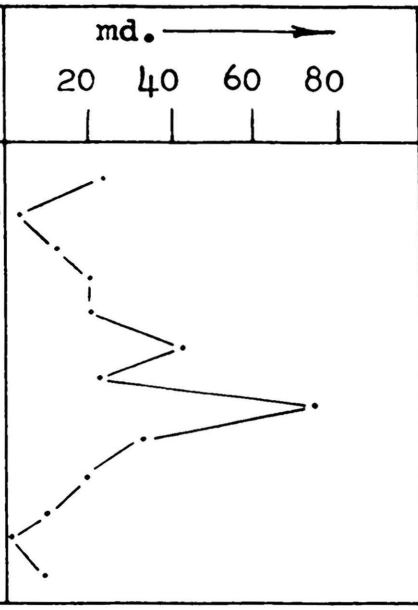
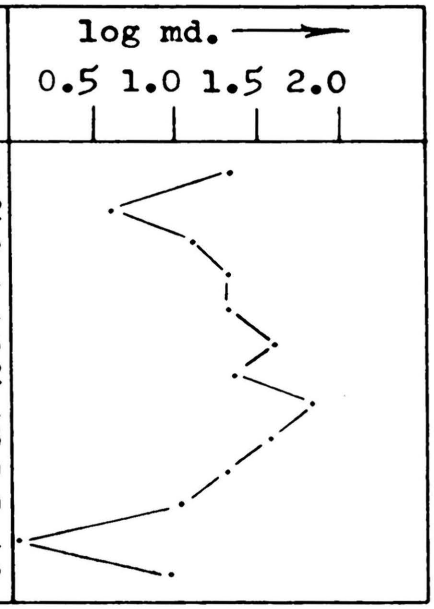
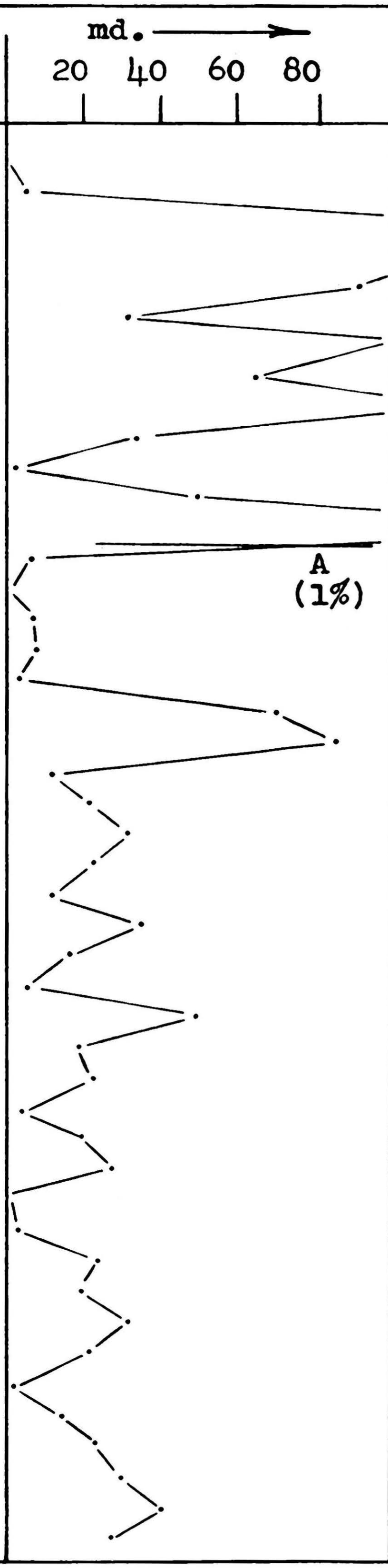
Depth (ft)	Perm.	Log Perm.
1753	24	1.380
1754	4.6	0.662
1755	14	1.146
1756	20	1.301
1757	20	1.301
1758	43	1.633
1759	23	1.362
1760	76	1.881
1761	33	1.518
1762	19	1.279
1763	10	1.000
1764	1.1	0.041
1765	9.7	0.986

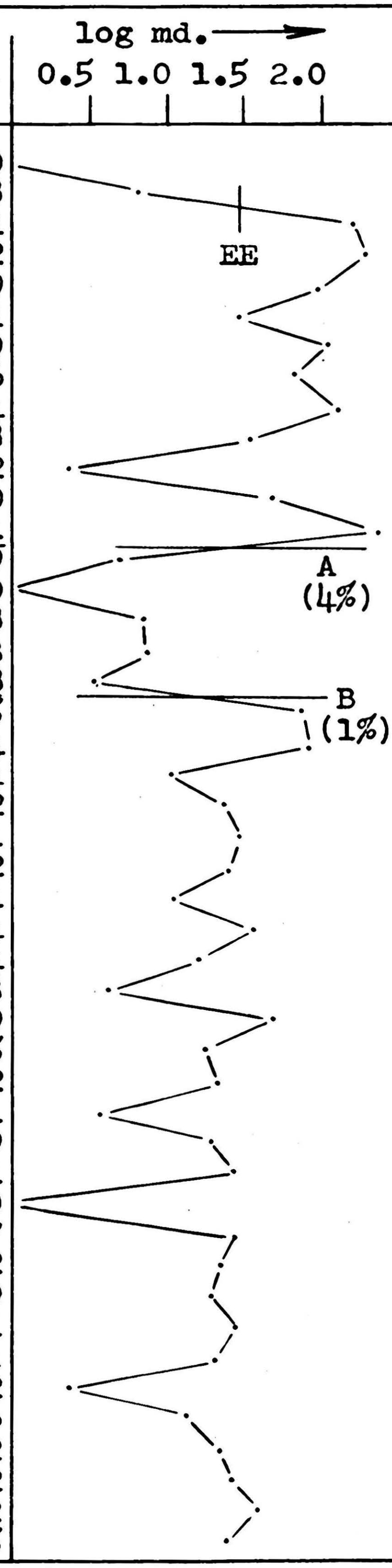




Well 1-Q-11

Depth (ft)	Perm.	Log Perm.
1698	0.2	0.000
1699	6.0	0.778
1700	159	2.201
1701	183	2.262
1702	91	1.959
1703	31	1.491
1704	107	2.029
1705	64	1.806
1706	127	2.104
1707	33	1.518
1708	2.2	0.342
1709	49	1.690
1710	252	2.401
1711	5.0	0.699
1712	0.9	0.000
1713	6.0	0.778
1714	6.5	0.813
1715	3.3	0.518
1716	70	1.845
1717	86	1.934
1718	11	1.014
1719	21	1.322
1720	31	1.491
1721	22	1.342
1722	11	1.041
1723	35	1.544
1724	16	1.204
1725	4.5	0.653
1726	50	1.699
1727	18	1.255
1728	22	1.342
1729	3.9	0.591
1730	19	1.279
1731	27	1.431
1732	0.6	0.000
1733	2.8	0.447
1734	23	1.362
1735	19	1.279
1736	31	1.491
1737	20	1.301
1738	2.3	0.362
1739	14	1.146
1740	22	1.342
1741	29	1.462
1742	40	1.602
1743	26	1.415





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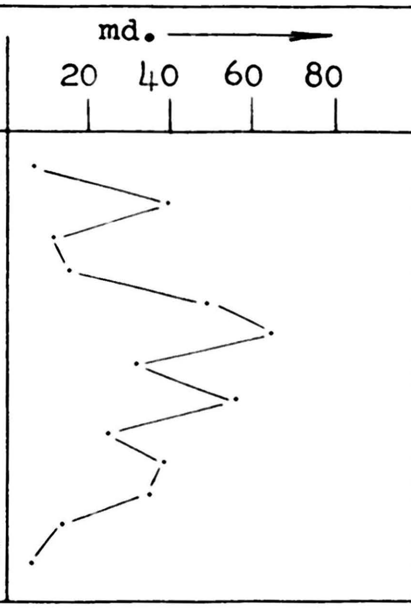
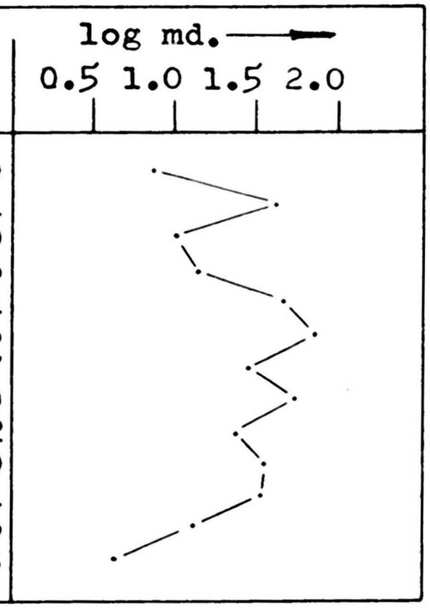
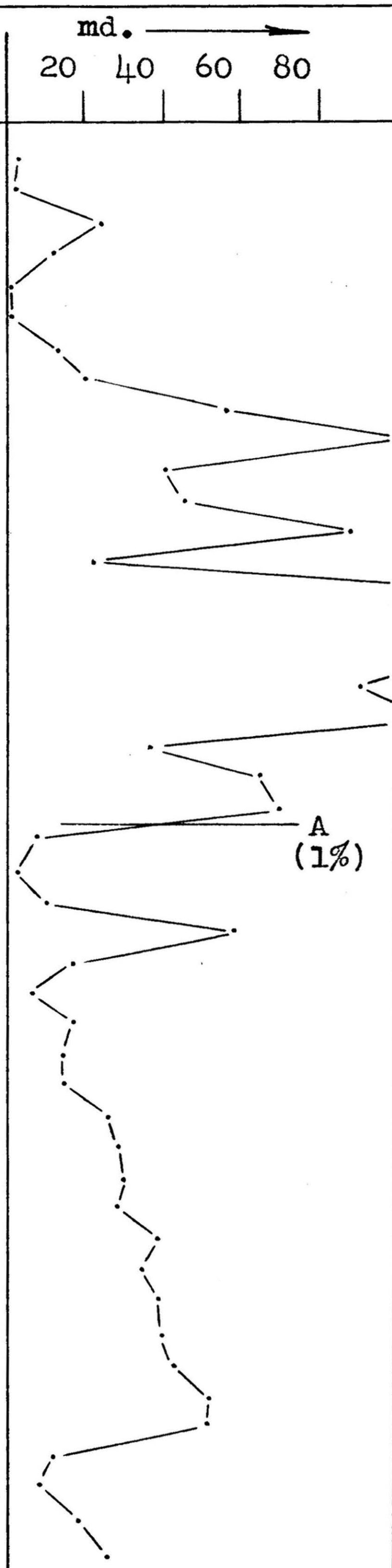
Depth (ft)	Perm.	Log Perm.
1744	6.3	0.799
1745	39	1.591
1746	10	1.000
1747	14	1.146
1748	48	1.681
1749	64	1.806
1750	30	1.477
1751	55	1.740
1752	23	1.362
1753	38	1.580
1754	34	1.531
1755	14	1.146
1756	4.5	0.653

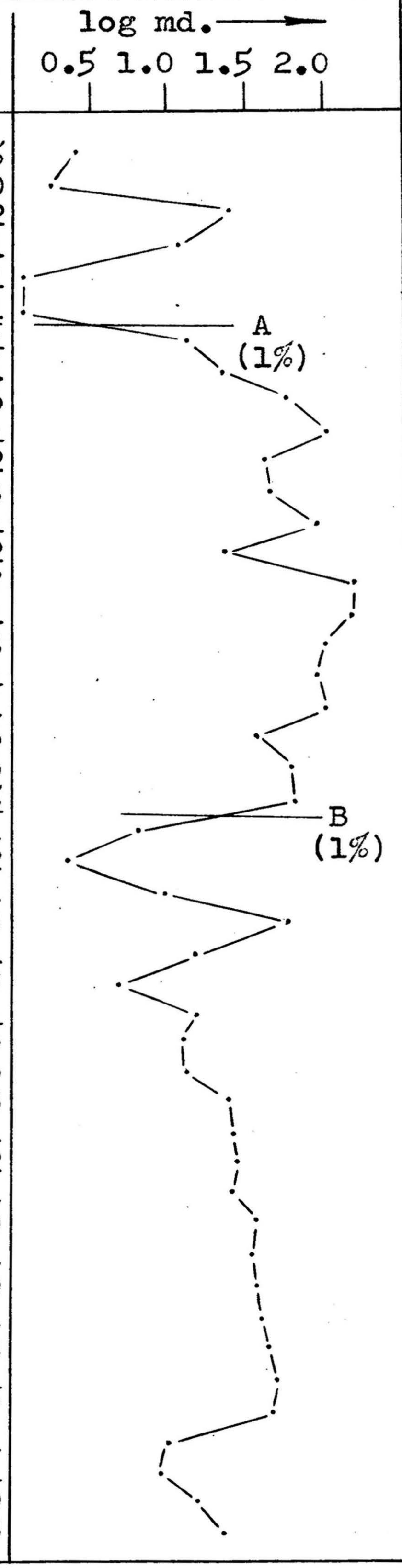




Well 1-0-11

Depth (ft)	Perm.	Log Perm.
1712	2.6	0.415
1713	1.7	0.230
1714	23	1.362
1715	11	1.041
1716	1.1	0.041
1717	1.1	0.041
1718	13	1.114
1719	20	1.301
1720	57	1.756
1721	101	2.004
1722	40	1.602
1723	45	1.653
1724	90	1.954
1725	22	1.342
1726	161	2.207
1727	160	2.204
1728	121	2.083
1729	92	1.964
1730	110	2.041
1731	36	1.556
1732	64	1.806
1733	70	1.845
1734	7.1	0.851
1735	2.1	0.322
1736	9.8	0.991
1737	58	1.763
1738	16	1.204
1739	5.1	0.707
1740	16	1.204
1741	14	1.146
1742	14	1.146
1743	25	1.398
1744	27	1.431
1745	29	1.462
1746	27	1.431
1747	38	1.580
1748	34	1.531
1749	38	1.580
1750	39	1.591
1751	42	1.623
1752	53	1.724
1753	51	1.707
1754	11	1.041
1755	8.2	0.914
1756	17	1.230
1757	25	1.398





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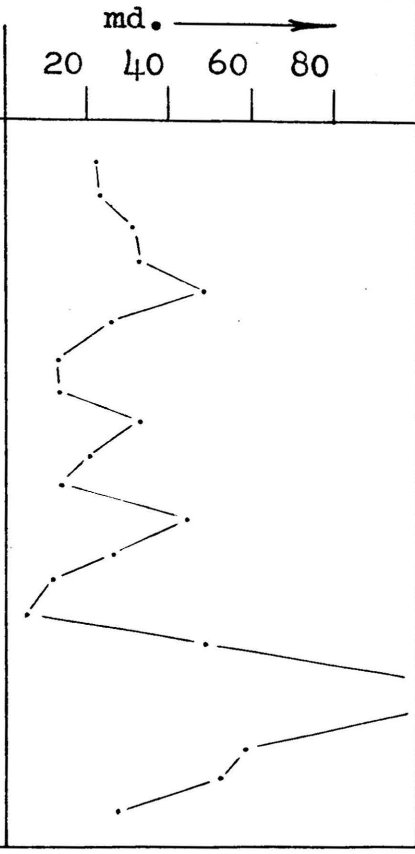
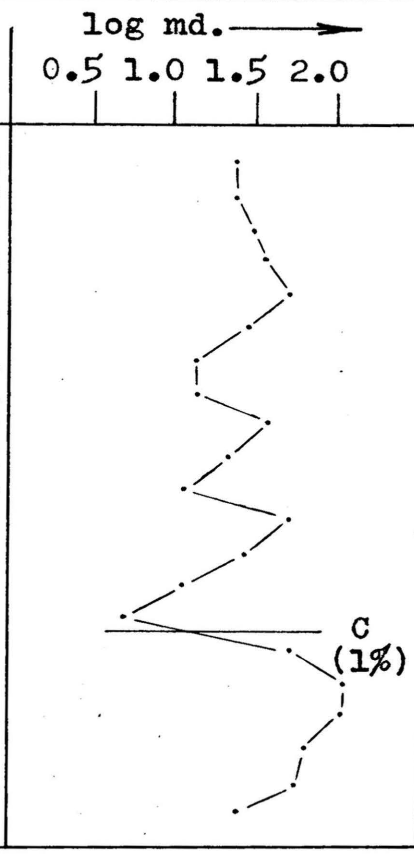
Depth (ft)	Perm.	Log Perm.
1758	22	1.342
1759	23	1.362
1760	31	1.491
1761	33	1.518
1762	49	1.690
1763	26	1.415
1764	13	1.114
1765	13	1.114
1766	33	1.518
1767	21	1.322
1768	14	1.146
1769	45	1.653
1770	27	1.431
1771	12	1.079
1772	4.9	0.690
1773	50	1.699
1774	114	2.057
1775	110	2.014
1776	59	1.771
1777	53	1.724
1778	28	1.447





APPENDIX B

Sample Calculations Performed Upon

Well Profile K-17

SECTION 1-B

Histogram and Cumulative Percent Curve Data and Calculations for Profile K-17 and Segments

Profile K-17:

Ordered Perm. Observations (md.)	Cumulative Sum	Cumulative Percent
0.8	0.8	0.1
0.9	1.7	0.2
1.5	3.2	0.4
1.9	5.1	0.6
2.0	7.1	0.8
2.4	9.5	1.1
2.6	12.1	1.4
2.7	14.8	1.7
3.4	18.2	2.1
4.0	22.2	2.6
4.8	27.0	3.1
4.9	31.9	3.7
6.1	38.0	4.4
6.2	44.2	5.1
6.3	50.5	5.9
6.8	57.3	6.6
9.3	66.6	7.7
12	78.6	9.1
13	91.6	10.6
15	106.6	12.4
17	123.6	14.3
19	142.6	16.5
20	162.6	18.8
23	185.6	21.7
25	210.6	24.4
27	237.6	27.6
31	268.6	31.2
32	300.6	34.9
33	333.6	38.7
36	369.6	42.9
43	412.6	46.7
49	461.6	53.5
51	512.6	59.5
63	575.6	66.9
81	656.6	76.2
96	752.6	87.3
109	861.6	100.0

Ordered Perm. Observations (md.)	Cumulative Sum	Cumulative Percent
Segment 1:		
0.9	0.9	0.1
1.5	2.4	0.4
1.9	4.3	0.7
2.0	6.3	1.0
2.4	8.7	1.4
2.6	11.3	1.8
2.7	14.0	2.3
4.0	18.0	2.9
6.3	24.3	3.9
6.8	31.1	5.0
13.0	44.1	6.8
19.0	63.1	10.2
23.0	86.1	13.9
25.0	111.1	17.9
33.0	144.1	23.2
36.0	180.1	29.0
43.0	223.1	35.9
49.0	272.1	43.8
63.0	335.1	54.0
81.0	416.1	67.0
96.0	512.1	82.4
109.0	621.1	100.0
Segment 2:		
0.8	0.8	0.3
3.4	4.2	1.8
4.8	9.0	3.7
4.9	13.9	5.8
6.1	20.0	8.3
6.2	26.2	10.9
9.3	35.5	14.8
12.0	47.5	19.8
15.0	62.5	26.0
17.0	79.5	33.1
20.0	99.5	41.4
27.0	126.5	52.6
31.0	157.5	65.5
32.0	189.5	78.5
51.0	240.5	100.0

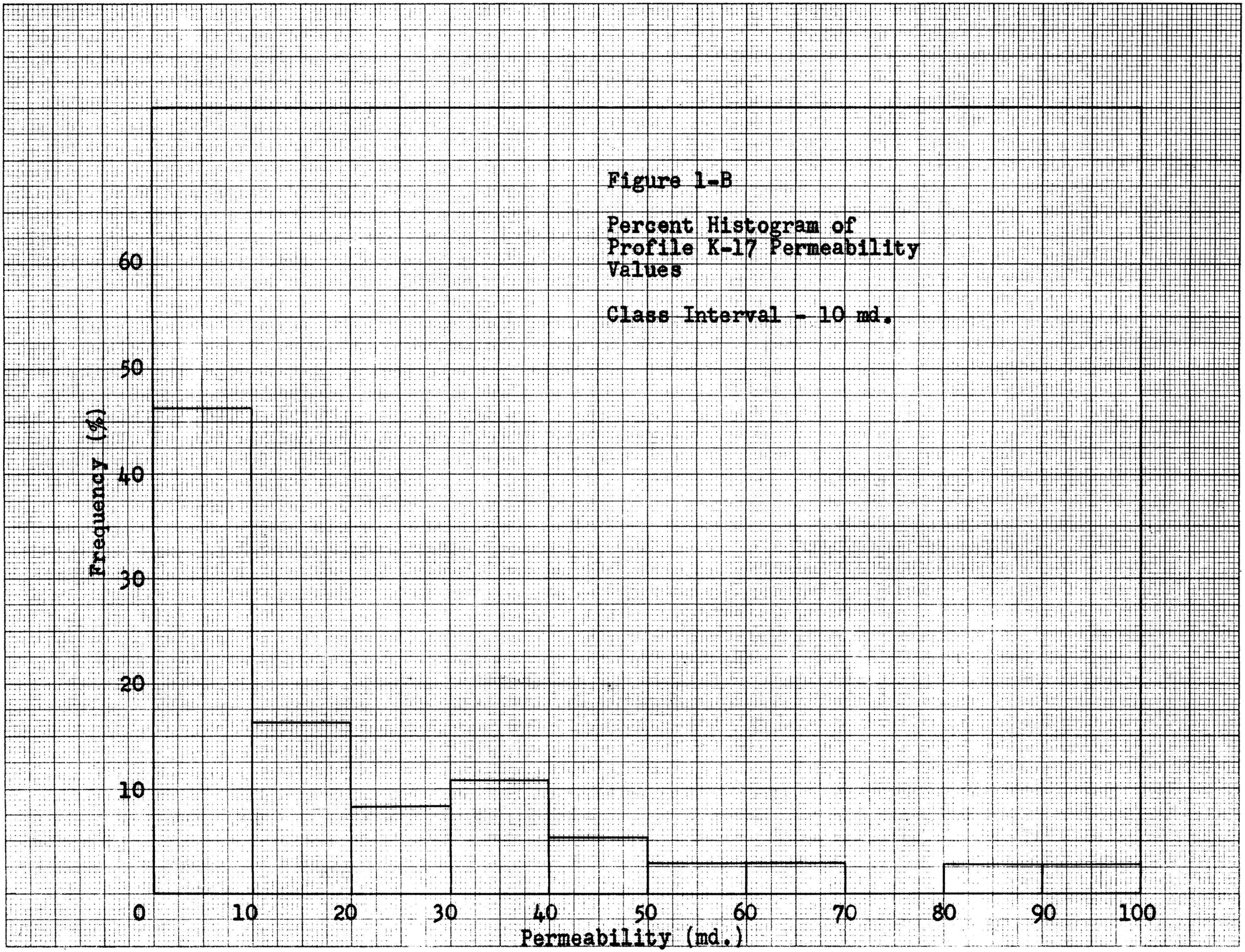


Figure 1-B
Percent Histogram of Profile K-17 Permeability Values
Class Interval - 10 md.
Frequency (%)
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
Permeability (md.)

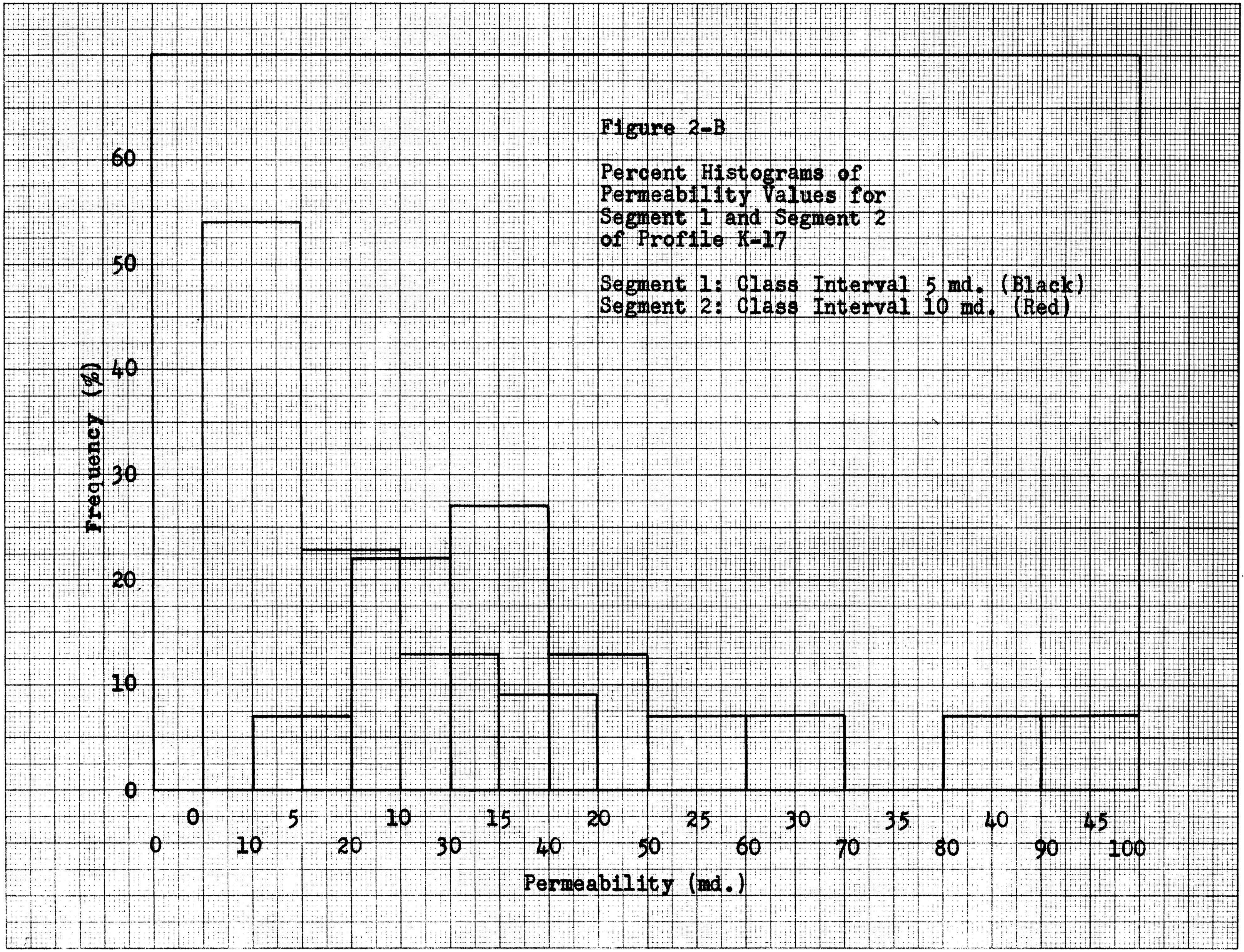


Figure 2-B
Percent Histograms of Permeability Values for Segment 1 and Segment 2 of Profile K-17
Segment 1: Class Interval 5 md. (Black)
Segment 2: Class Interval 10 md. (Red)
Frequency (%)
0
10
20
30
40
50
60
0
5
10
15
20
25
30
35
40
45
0
10
20
30
40
50
60
70
80
90
100
Permeability (md.)

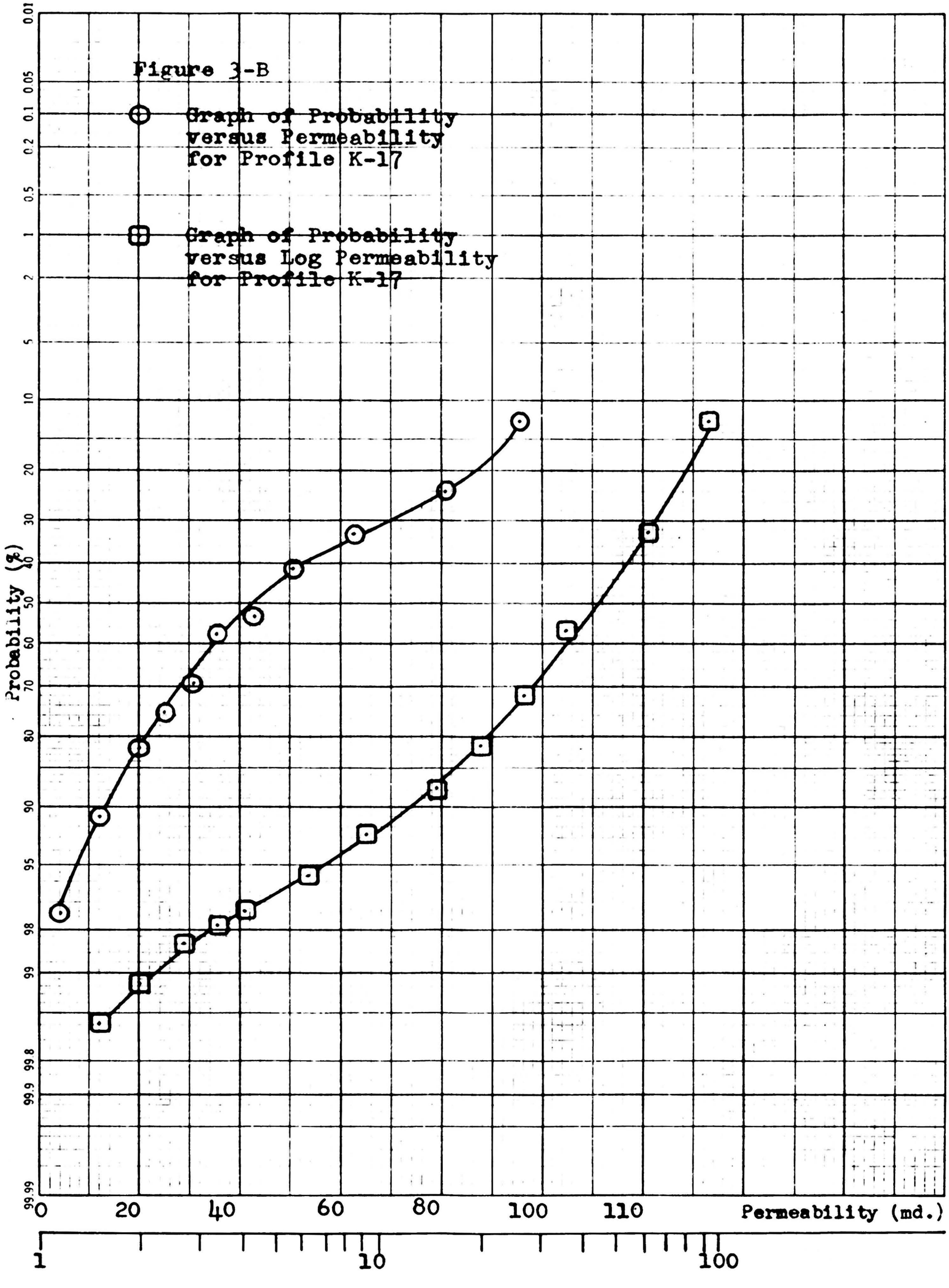


Figure 3-B
Graph of Probability versus Permeability for Profile K-17
Graph of Probability versus Log Permeability for Profile K-17
Probability (%)
0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
99.99
0
20
40
60
80
100
110
Permeability (md.)
1
10
100

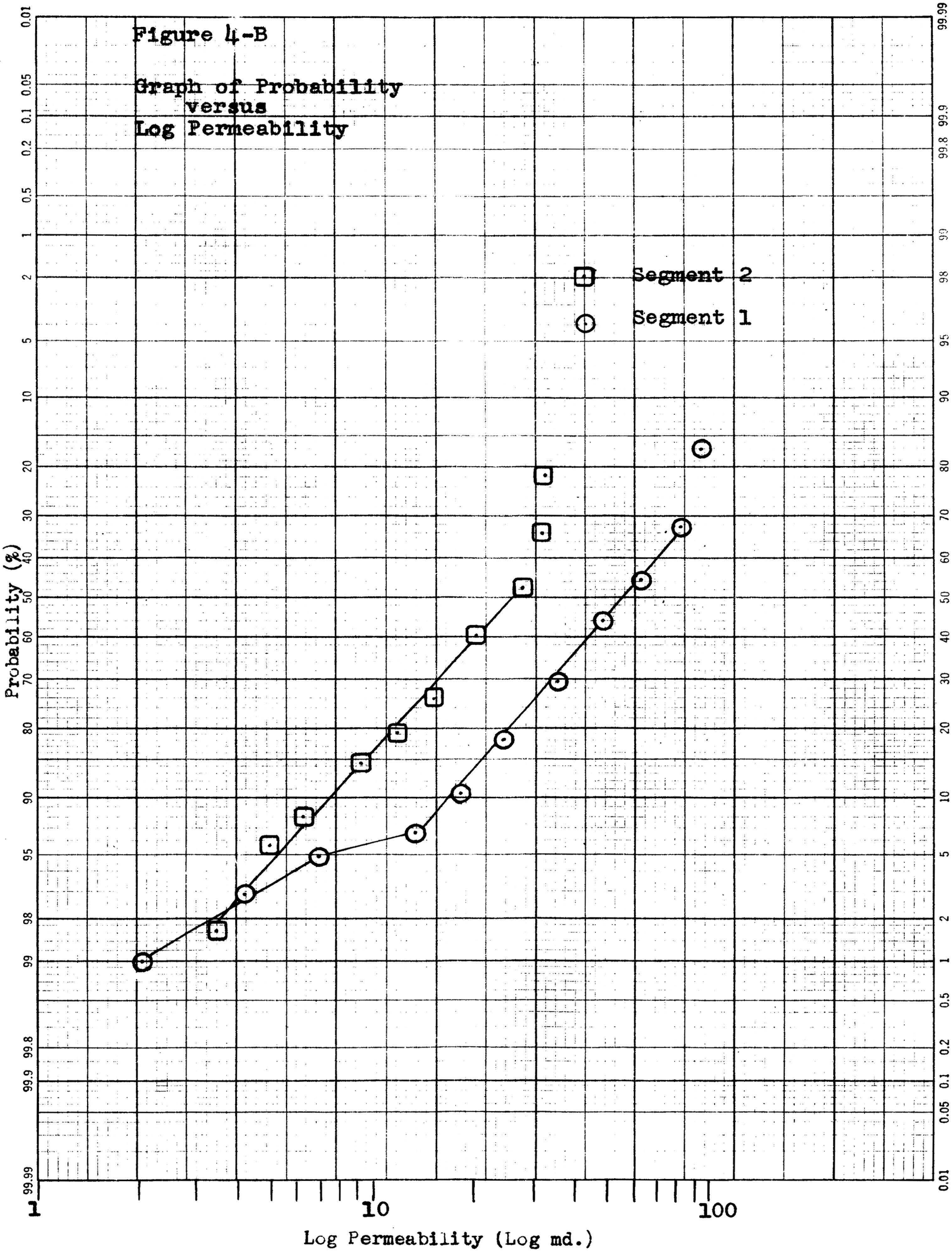


Figure 4-B
Graph of Probability
versus
Log Permeability
Segment 2
Segment 1
Probability (%)
Log Permeability (Log md.)
1
10
100
0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
99.99

SECTION 2-B

Calculation of First and Second Cumulants for Segments 1 and 2 of Profile K-17

Segment 1:

$N = 22$

$\sum x = 621.1$

$\sum x^2 = 40,078.4$

$\sum x^3 = 3,278,548.5$

$\sum x^4 = 279,775,122.2$

$K_2 = 1074$

$g_1 = 0.8$ (5% level)

$g_2 = 1.5$ (5% level)

$$K_3 = (N^2S_3 - 3nS_2S_1 + 2S_1^3)/N(N - 1)(N - 2)$$

$$= 45,890$$

$$g_1 = K_3K^{3/2} = 1.2$$

$$K_4 = \left[N(N + 1)S_4 - 3(N - 1)S_2^3\right]/(N - 1)(N - 2)(N - 3)$$

$$= 6,200,330$$

$$g_2 = K_4/K_2^2 = 5.4$$

Segment 2:

$N = 15$

$\sum x = 240.5$

$\sum x^2 = 6,594.4$

$\sum x^3 = 235,446.7$

$K_3 = 3450$

$g_1 = 1.3$

$K_4 = 218,154$

$g_2 = 15.3$

$\sum x^4 = 9,595,159.1$

$K_2 = 196$

$g_1 = 1$ (5% level)

$g_2 = 2$ (5% level)

SECTION 3-B

Zonation Calculations
Profile K-17

Profile K-17:

n	Log Data	Cumulative Sum	Cum. Sum of Squared Data	n(N-n)/N	V	
1	0.602	0.602	0.362	0.972	0.206	
2	0.176	0.778	0.393	1.892	0.924	
3	0.380	1.158	0.538	2.757	1.439	
4	0.431	1.589	0.724	3.568	1.911	
5	1.799	3.388	3.960	4.324	0.802	
6	2.037	5.425	8.109	5.027	0.152	A
7	0.279	5.704	8.187	5.676	0.477	
8	0.301	6.005	8.278	6.270	0.915	
9	0.000	6.005	8.278	6.811	1.742	
10	0.415	6.420	8.450	7.297	2.281	
11	0.799	7.219	9.088	7.729	2.427	
12	1.556	8.775	11.510	8.108	1.804	
13	1.398	10.173	13.464	8.432	1.434	
14	1.279	11.452	15.100	8.703	1.212	
15	0.833	12.285	15.793	8.919	1.346	
16	1.908	14.193	19.434	9.081	0.748	
17	1.114	15.307	20.675	9.189	0.704	
18	1.633	16.940	23.342	9.243	0.416	
19	1.982	18.922	27.270	9.243	0.114	
20	1.362	20.284	29.125	9.189	0.056	
21	1.690	21.974	31.981	9.081	0.001	
22	1.519	23.493	34.289	8.919	0.017	
23	1.230	24.723	35.801	8.703	0.107	
24	1.176	25.899	37.184	8.432	0.058	B
25	0.531	26.430	37.466	8.108	0.260	
26	1.491	27.921	39.690	7.729	0.050	
27	0.690	28.611	40.166	7.297	0.009	
28	0.792	29.403	40.793	6.811	0.000	
29	0.785	30.188	41.409	6.270	0.011	
30	1.301	31.489	43.102	5.676	0.000	
31	1.079	32.568	44.266	5.027	0.000	
32	0.681	33.249	44.730	4.324	0.042	
33	1.708	34.957	47.647	3.568	0.026	
34	0.968	35.925	48.584	2.757	0.018	
35	1.505	37.430	50.849	1.892	0.244	
36	1.431	38.861	52.897	0.972	1.158	
37	0.000	38.861	52.897			

Analysis of Variance

	D.F.	Sum of Squares	Mean Square	F
Total	36	12.081		
Between	1	2.430	2.430	9.067 (1%)
Within	36	9.651	0.268	

Segment A:

n	n(N-n)/N	V
1	0.909	0.003
2	1.636	0.174
3	2.182	0.301
4	2.545	0.421
5	2.727	0.004
6	2.727	0.813
7	2.545	0.486
8	2.182	0.263
9	1.636	0.006
10	0.909	0.022

Analysis of Variance:

	D.F.	Sum of Squares	Mean Square	F
Total	11	3.371		
Between	1	0.811	0.811	3.48 (NS)
Within	11	2.560	0.233	

Segment B:

n	Cumulative Sum	n(N-n)/N	V
1	1.556	0.962	0.119
2	2.945	1.846	0.146
3	4.233	2.654	0.128
4	5.066	3.385	0.012
5	6.974	4.038	0.196
6	8.088	4.615	0.134
7	9.721	5.115	0.282
8	11.703	5.538	0.697
9	13.065	5.885	0.758
10	14.755	6.154	1.086
11	16.274	6.346	1.313
12	17.504	6.462	1.301
13	18.680	6.500	0.436
14	19.211	6.462	0.417
15	20.702	6.346	0.144
16	21.392	6.154	0.246
17	22.184	5.885	0.380
18	22.969	5.538	0.204
19	24.270	5.115	0.257
20	25.349	4.615	0.221
21	26.030	4.038	0.055
22	27.738	3.385	0.275
23	28.706	2.654	0.193
24	30.211	1.846	0.545
25	31.642	0.962	1.540
26	31.642		

Analysis of Variance:

	D.F.	Sum of Squares	Mean Square	F
Total	25	5.300		
Between	1	1.124	1.124	6.73 (2%)
Within	25	4.176	0.167	

APPENDIX C

Single Grouping Randomized Block Design

Table form of data:

Column

Row	1	2	3	j	c	Row Means
1	x_{11}	x_{12}	x_{13}	x_{ij}	x_{1c}	$x_{1.}$
2	x_{21}	x_{22}	x_{23}	x_{2j}	x_{2c}	$x_{2.}$
i	x_{i1}	x_{i2}	x_{i3}	x_{ij}	x_{ic}	$x_{i.}$
r	x_{r1}	x_{r2}	x_{r3}	x_{rj}	x_{rc}	$x_{r.}$
Column Means	$x_{.1}$	$x_{.2}$	$x_{.3}$	$x_{.j}$	$x_{.c}$	$x_{..}$

Analysis of variance table:

Variation	Degrees of Freedom	Sum of Squares	Mean Square
Total	rc-1	$\sum(x_{ij}-x_{..})^2$	
Between Row Means	r-1	$\sum(x_{i.}-x_{..})^2$	Sum of Squares divided by the Degrees of Freedom
Between Column Means	c-1	$\sum(x_{.j}-x_{..})^2$	
Residual	(r-1)(c-1)	$\sum(x_{ij}-x_{i.}-x_{.j}+x_{ii})^2$	

Computations:

Total Sum of Squares = $\sum x_{ij}^2 - x_{..}^2/rc$

Row Sum of Squares $=\sum x_{i.}^2/c - x_{..}^2/rc$

Column Sum of Squares = $\sum x_{.j}^2/r - x_{..}^2/rc$

Residual Sum of Squares = Total - Row & Column Sum of Squares

Calculation of missing data:

A single missing unit may be replaced by

$$(cB + rT - G)/(c-1)(r-1)$$

where B is the total of the remaining units in the column where the missing unit appears, T is the total of the units in the row where the missing unit appears, and G is the grand total.

For several missing units Cochran and Cox (3, p. 111) suggest the following procedure: For missing units a, b, c, d,..., first estimate values for all units except a. The above formula is then used to find an approximation of a. With this approximation and the values previously assumed for c, d,..., the above formula is used to approximate b. After a complete cycle of these operations, a second approximation is found for a and so on until the new approximations are not materially different from those found previously.

For each missing unit, one degree of freedom is subtracted from the total and residual sum of squares.

APPENDIX D

Analysis of Major Zones for
Significant Variations

Zone 1:

1.810	
1.733	
1.777	1.583

The value 1.583 can be shown to be significantly different from the other three numbers and as such should not be included in Zone 1. The remaining three values have a standard deviation of 0.0388 which indicates, when compared with the mean of 1.773, good uniformity.

Zone 2:

			Row Totals
	1.347	1.479	2.826
	1.422	1.475	2.897
	a	1.395	1.395
	1.314	1.262	2.576
Column Totals	4.083	5.611	9.694

Calculation of Missing Value:

$$a = cB - rT - G \quad /(c-1)(r-1)$$

$T = 1.395$

$G = 9.694$

$B = 4.083$

$c = 2$

$r = 4$

$\therefore\ a = 1.351$

Analysis of Variance

	D.F.	Sum of Squares	Mean Square	F
Total	6	0.041		
Column	3	0.004	0.0013	1 (NS)
Row	1	0.028	0.028	6.22 (NS)
Residual	2	0.009	0.0045	

Zone 3:

			Row Totals
	0.871	1.023	1.894
	1.054	1.135	2.189
	1.138	b	1.138
Column Totals	2.063	2.158	5.221

Calculation of Missing Value:

B 2.158

T 1.138

G 5.221

c 2

r 3

∴ b 1.255

Analysis of Variance:

	D.F.	Sum of Squares	Mean Square	F
Total	4	0.084		
Column	2	0.020	0.010	10 (NS)
Row	1	0.063	0.063	63 (NS)
Residual	1	0.001	0.001	

Figure 1

Plan View of Example Wells

Olympic Pool
Hughes County, Oklahoma
Scale: 1" = 400'

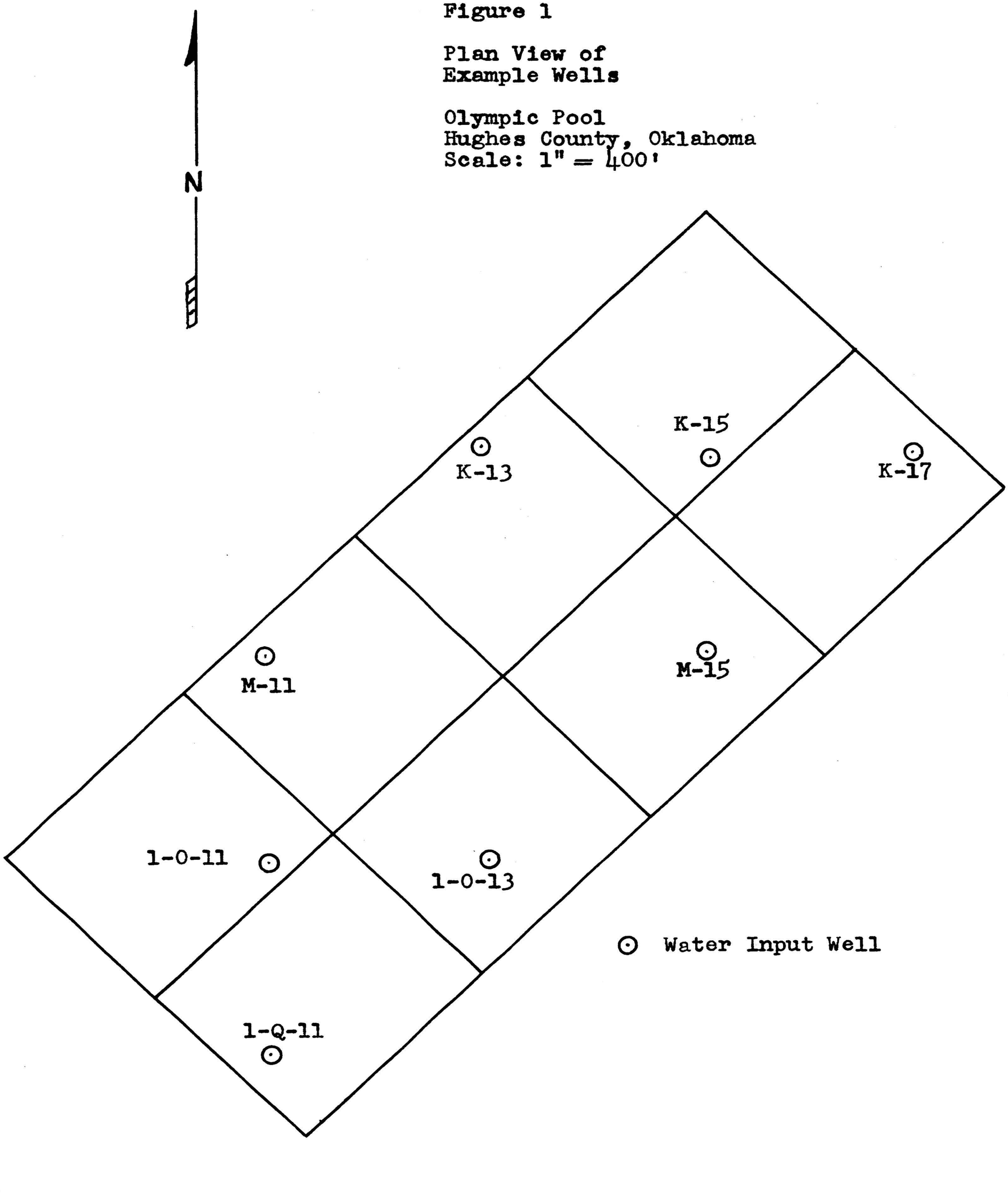


Figure 2
Geologic Cross Section
Showing Continuity of
Permeability Values
Within the Example Wells

(Not to Scale)

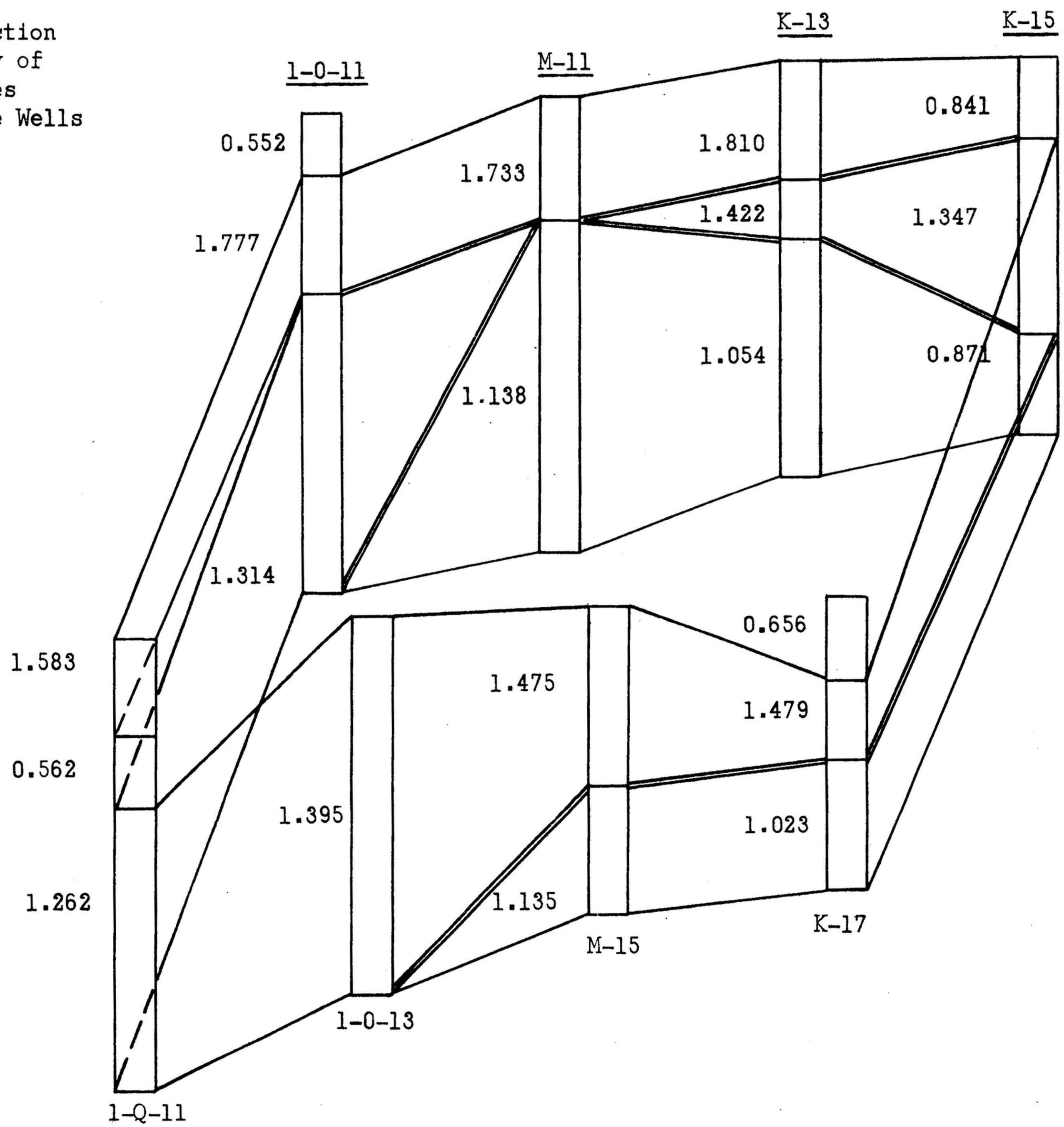


FIGURE 3

Block Diagrams Illustrating, for Each Major Zone, the Means and Standard Deviation of the Log Permeability Observations Found Therein

Form of Block Data
Well No. Mean Standard Deviation

Zone 1

K-15	K-17
K-13 1.810 0.933	M-15
M-11 1.733 0.545	1-0-13
1-0-11 1.777 0.319	1-Q-11 1.583 0.757

Zone 2

K-15 1.347 0.444	K-17 1.479 0.332
K-13 1.422 0.356	M-15 1.475 0.496
M-11	1-0-13 1.395 0.351
1-0-11 1.314 0.316	1-Q-11 1.262 0.406

Zone 3

K-15 0.871 0.546	K-17 1.023 0.451
K-13 1.054 0.444	M-15 1.135 0.391
M-11 1.138 0.363	1-0-13
1-Q-11	1-Q-11

TABLE 4

Tabulation of Means, Standard Deviations, and Observation Numbers for each Permeability Profile and each Profile Segment.

	Mean	Standard Deviation	Number
Well K - 17	23.29	27.19	37
Segment 1	28.23	32.77	22
Segment 2	16.03	13.99	15
Well K-15	22.63	19.90	50
Segment 1	27.48	21.42	33
Segment 2	13.22	12.33	17
Well M-15	34.65	30.04	42
Segment 1	43.89	33.05	27
Segment 2	18.03	12.39	15
Well K-13	33.16	34.62	56
Segment 1	76.79	41.12	14
Segment 2	18.62	12.84	42
Well 1-0-13	38.86	38.07	58
Segment 1	53.05	50.67	26
Segment 2	27.33	14.11	32
Well M-11	38.15	59.47	60
Segment 1	104.58	107.65	11
Segment 2	21.56	5.66	49
Well 1-Q-11	37.00	46.80	59
Segment 1	84.95	77.43	13
Segment 2	23.45	19.29	46
Well 1-0-11	39.25	36.40	67
Segment 1	56.48	50.59	22
Segment 2	30.38	23.47	45

TABLE 5
Tabulation of Means, Standard Deviations, and Observation Numbers for the Significantly Distinct Segments of each Well Profile of Transformed Data.

	Mean	Standard Deviation	Number
Well K-17			
Segment 1	0.655	0.659	11
Segment 2	1.479	0.332	11
Segment 3	1.023	0.451	15
Well K-15			
Segment 1	0.841	0.678	9
Segment 2	1.347	0.444	24
Segment 3	0.871	0.546	17
Well M-15			
Segment 1	1.475	0.496	27
Segment 2	1.135	0.391	15
Well K-13			
Segment 1	1.810	0.933	14
Segment 2	1.422	0.356	6
Segment 3	1.054	0.444	36
Well 1-0-13			
Segment 1	1.395	0.351	58
Well M-11			
Segment 1	1.733	0.545	13
Segment 2	1.138	0.363	47
Well 1-Q-11			
Segment 1	1.583	0.757	13
Segment 2	0.562	0.336	5
Segment 3	1.262	0.406	41
Well 1-0-11			
Segment 1	0.552	0.554	6
Segment 2	1.777	0.319	16
Segment 3	1.314	0.316	40
Segment 4	1.803	0.265	5

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VITA

LeRoy Allan Beghtol, son of Roy and Jewel Beghtol, was born February 13, 1935 in Chicago, Illinois. At the age of five his family moved to Lombard, Illinois where he received his elementary and secondary education.

He enrolled at the Missouri School of Mines and Metallurgy in September 1952 and was graduated in June 1956 with the degree of Bachelor of Science in Mining Engineering - Mining Geology, Petroleum Option.

He entered the Graduate School of the same institution in September 1956, at which time he was granted a Graduate Assistantship.

During the summers of 1955 and 1957 he was employed respectively in exploration geology and petroleum engineering research. For the spring semester of 1957 he was employed as a part-time instructor in Mining Engineering.